

Spend your decennial age: a rule of thumb for retirement

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Abstract

In Australia, the majority of people retire with lump sum benefits from defined contribution superannuation schemes, and elect to take their benefits either in the form of Account-Based Pensions (ABPs) or lump sums.[1] Determining the appropriate drawdown from assets in retirement is a complicated and important problem faced by many older Australians. That complexity and importance mean that preparation of comprehensive, independent advice covering retirement drawdown is critical for the majority who do not necessarily have the financial literacy to navigate the decisions themselves. Advice is often expensive and therefore not a feasible option for everyone.

This paper addresses the needs of retirees who are entitled to full or part pension, and who usually do not commission comprehensive, independent advice on drawdowns, but rely on other inputs, such as the ABP minimum withdrawal rates covering pension assets in the superannuation system.

The purposes of this paper are to develop some ABP drawdown rules and to investigate how alternative drawdown rules affect pensioner welfare as determined by using a utility metric, taking into account the lifetime interactions with the Age Pension means tests for different asset balances. We use the legislated ABP minimum drawdown rates as a starting point and explore some previously published drawdown rules. Then, using the results of dynamic programming calculations that produce optimum drawdown rates by age and asset balance, we develop new drawdown rules, including a simple rule of thumb, that yield improved total lifetime utility of consumption for retirees.

Keywords: Retirement spending, asset drawdown, account-based pension, consumption, longevity risk, age pension means tests, rule of thumb, utility curves, risk aversion, ABP minimum withdrawal rates.

[1]The retirement product environment may change with the introduction of the Retirement Covenant and the development of CIPRs (Comprehensive Income Products for Retirement).

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1 Introduction

Retirees in Australia, many of whom may not have a high degree of financial literacy, face a complex but important problem. The great majority of retirees rely on an Account Based Pension (ABP) as the financial product that will fund their retirement spending. These retirees bear many risks, including but not limited to: investment risk (including sequencing risk), longevity risk, real rate risk, inflation risk and regulatory risk. They need to decide how much of their ABP they can afford to spend each year, in order to provide as attractive a lifestyle as possible, but without taking so much risk that, at a later stage, their assets will have diminished and be inadequate to meet their lifestyle wishes. For the time being, for the majority of retirees who do not obtain formal financial advice, no guidance (outside the academic literature) is provided on this very important topic, and many fall back on the statutory minimum ABP withdrawal rates. It is important to note that the legislated ABP minimum withdrawal rates were never designed to optimise retirement outcomes, and (as we show) drawing down from the ABP at these rates is not usually optimal.

The total volume of assets in superannuation schemes is currently about $2.8 trillion of which roughly 30% is in pension phase. The question of how much to draw down from the retiree's ABP is an important issue and we believe that ABP account holders are likely to appreciate guidance from a well-informed, independent source. For the actuarial profession, it is "time to act" to provide guidance to unadvised retirees confronted with this extremely challenging problem.

This paper proceeds as follows.

In Section 2, the features of the Australian retirement income system are described. We note the current push towards retirement products with at least partial inbuilt longevity protection. We also note the significance of the means testing arrangements for the Age Pension, especially the assets test.

In Section 3, we consider many aspects of a retiree's individual personal circumstances that could be regarded as relevant to the drawdown decision. A key objective of this paper is to produce a "rule of thumb" which will make it easier for retirees to determine how much they should spend in the current year. By its very nature, a rule of thumb ideally should be as simple as possible, but there are also competing objectives, namely that the rule should be as general as possible and demonstrably reasonable, or at least not unreasonable. There would be no point in producing a rule which is very simple, but which results in retirees being guided towards spending either an unreasonably low, or an unreasonably high, amount from their ABP each year. In Section 3, we describe our approach to the many relevant differences in people's circumstances and, in particular, we state the assumptions we have made to keep the problem from becoming too multi-dimensional and therefore unmanageable.

In Section 4, we introduce "consumption utility curves" as a means of comparing and contrasting the retirement outcomes of alternative drawdown (spending)

rules. In particular, we introduce MDUF[2], the Members' Default Utility Function, which provides the software we have used in this paper to "score" alternative spending rules. We then discuss how we have elected to parameterise the many input requirements of MDUF and describe the output measures that MDUF provides.

In Section 5, we describe five potential spending rules that have been identified in previous literature, namely the anecdotal "spend the income" rule, the Bengen (1994) 4% rule, the New Zealand Society of Actuaries (NZ SOA) 6% rule[3], the NZ SOA "life expectancy rule"[4] and the ABP minimum drawdown rule. The first of these rules is not appropriately specified for evaluation using MDUF but we have applied MDUF to the latter four rules and present the relevant comparisons.

In Section 6, we apply dynamic programming techniques to identify the optimal drawdown as a function of age and ABP balance, and use this analysis to develop three alternative rules including a simple "rule of thumb". We show that each of these three rules yields better lifetime utility compared to drawing down at the statutory minimum rate.

In Section 7, we provide the sensitivity analysis results to test the robustness of the derived rules of thumb in comparison to the statutory minimum drawdown rules and the optimal drawdown rules. To achieve this, we apply the rules to both genders, and test alternative assumptions about the assumed mean asset return and asset volatility.

Section 8 discusses the findings and lists further research that would be useful to support the development and implementation of indicative spending guidance for retirees.

Section 9 concludes the paper.

[2] http://membersdefaultutilityfunction.com.au/
[3] Retirement Incomes Interest Group of the New Zealand Society of Actuaries (2017).
[4] Retirement Incomes Interest Group of the New Zealand Society of Actuaries (2017).

2 Spending decisions in retirement – current Australian environment

2.1 Three pillars

In Australia, consumption expenditure after a person ceases working is typically funded from one of three sources i.e. the three pillars:

- The full or part Age Pension;
- Occupational superannuation arrangements; and
- The retiree's own financial and investment assets outside the superannuation system.

The entitlement or potential entitlement to the Age Pension is an extremely important feature of the Australian retirement incomes system. The full Age Pension payable to a single person as at the date when they first attain the eligibility age has an actuarial present value in excess of $500,000, and the full Age Pension payable to a couple attaining eligibility age has an actuarial value in excess of $800,000 (Rice, 2018).

In this paper we focus on those individuals who are NOT comfortably self-funded because:

1. Wealthier retirees can generally afford to pay for financial advice, which takes into account all their personal circumstances, and a higher proportion of wealthier retirees obtain advice than poorer retirees.

2. Wealthier retirees are more likely to have a significant bequest motive, and hence are likely not to draw down their assets fully. As the primary objective of the superannuation system is '*to provide income in retirement to substitute or supplement the Age Pension*[5]*,* this paper focuses on retirees who are more likely to spend down their assets in full.

Since 1992[6], it has been compulsory for all Australian employers to make contributions (termed the "Superannuation Guarantee Contributions" or "SG contributions") on behalf of all employees receiving remuneration above a certain minimum threshold (currently $450 before tax in a calendar month) to a superannuation fund nominated by the employee.

Retail and industrial superannuation funds generally have many investment choices from which members can choose, offering a wide variety of asset allocations to members. Where a member does not explicitly choose an option, the fund will default a new member into the fund's "MySuper"

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[5] https://www.legislation.gov.au/Details/C2016B00182

[6]https://www.aph.gov.au/About_Parliament/Parliamentary_Departments/Parliamentary_Library/pubs/BN/0910/ChronSuperannuation

investment option. Usually the MySuper investment option is "balanced"[7], meaning that between 55% and 80% of the assets within that particular option may be classified as "growth" assets and the remaining 20% to 45% as "defensive" assets. Some funds also provide "glide path" or life-cycle options that alter the growth-defensive mix as the member approaches retirement. Members can vary their choice of investment option within the fund at any time.

2.2 Age pension

An extremely significant feature of the Australian retirement funding environment is the Age Pension, which commenced in 1901[8]. Australian residents with at least 10 years' residency are eligible on attaining age pension eligibility age, currently 65.5 years, but scheduled to increase to 67 years with effect from 1/7/2023[9]. The amount of the age pension payable in Australia is relatively generous by international standards (Asher and De Ravin, 2016).

The age pension in Australia is means tested. Assuming that the applicant meets residency and age requirements, the actual amount of pension payable is the lower of the two amounts calculated under the Income Test and the Assets Test.

Currently, a single age pensioner can earn up to $4,472 per annum without being impacted by the Income Test[10]. Thereafter, each dollar of fortnightly income results in a 50 cents reduction in the fortnightly amount payable. Income earned from financial assets is not taken to be the actual income from those assets, but "deemed" income. Deemed income for a single is 1.75% of the first $51,200 of assets for a single plus 3.25% of the excess over $51,200[11]. For a couple, the threshold is $85,000 but the same deemed earnings rates apply both to the amount of financial assets within and in excess of the couple threshold.

Similarly, there is a lower Assets Test threshold such that, if a single pensioner has assets below this level, the amount of fortnightly age pension payable under the Assets Test is the maximum age pension payable. For assets above the threshold, the amount payable is reduced by $3 per fortnight per each additional $1000 of assets. This results in a "taper rate" of 7.8% - the $3 fortnightly reduction times 26 fortnights per annum results in a $78 per annum reduction for each $1000 of assets in excess of the Lower Asset Test Threshold.

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[7] https://www.superguide.com.au/comparing-super-funds/superannuation-investment-difference-balanced-growth-option

[8] See https://dictionaryofsydney.org/entry/health_and_welfare

[9] https://www.humanservices.gov.au/individuals/services/centrelink/age-pension/eligibility/age-rules

[10] https://www.humanservices.gov.au/individuals/enablers/income-test-pensions/30406

[11] See https://www.humanservices.gov.au/individuals/enablers/deeming

The age pension comprises the dominant portion of retirement funding for those who receive the full Age Pension[12] (currently $24,081.20 for individuals and $36,301.20 for couples), and also a significant part for many of those receiving a part pension as a result of the operation of the means tests.

In total, about two thirds (66%) of Australians who meet the age eligibility requirements for the Age Pension receive either a full or a part age pension (DSS 2016; ABS 2017). Modelling performed by actuarial consulting firm Rice Warner[13] shows that, over time, many people will increasingly self-fund their retirement and the proportion of the population receiving the pension will fall. By 2038, only 57 per cent of the eligible population will receive any age pension, with 29 per cent on the full pension and 27.5 per cent receiving a part pension.

An important consequence of the Assets Test is its implications for optimal consumption. Retirees impacted by the Assets Test have a clear incentive to spend down their assets more quickly because the faster they spend down those assets, the greater their future age pension entitlements[14].

2.3 Pension products in retirement

When members of a superannuation fund attain a "condition of release", they are able to withdraw their funds from the superannuation environment or to shift their accumulation account balance into pension phase. For the time being, the pension product environment in Australia is considered under-developed. Currently the great majority of those who convert their accumulation balances to pension phase, use their balances to purchase an "Account-Based Pension" (ABP), which is effectively a pension product where the pensioner has control over both asset allocation and the rate at which the pension is "drawn down", subject only to statutory minimum withdrawal rates specified in the Superannuation Industry (Supervision) ("SIS") legislation. Those minimum rates are shown in the following Table.

[12] https://www.humanservices.gov.au/individuals/services/centrelink/age-pension/eligibility/payment-rates

[13] See https://www.professionalplanner.com.au/2019/03/landscape-of-the-100-year-life/

[14] The Age Pension also provides longevity protection, assuming eligibility is maintained.

Table 1: Minimum statutory drawdown rates for ABPs

Age Bracket	Minimum Drawdown %
Below 65	4
65-74	5
75-79	6
80-84	7
85-89	9
90-94	11
95 or older	14

Recognising the under-developed state of pension products in Australia, the Government has introduced the "retirement covenant", which is effectively a requirement for superannuation funds to develop Comprehensive Income Products for Retirement (or CIPR). Such products are expected ultimately to include components of either immediate or deferred life annuities, although the "retirement covenant" does not mandate that these products must be guaranteed. The government intends to address one of the key problems of ABPs through the retirement covenant, namely the management of longevity risk. There appears to be an expectation that circa 25% of a typical CIPR may be annuitised in some form, especially in light of the most recent changes to the means test rule for pooled lifetime income streams[15].

Despite the imminence of CIPRs and their potential impacts, the authors believe that the key question of what percentage to draw from ABPs will remain critical for the near to medium future. Our reasons are:

1. It is not yet clear how quickly or enthusiastically superannuation funds will develop or promote their CIPRs.

2. It is not clear to what extent retirees will adopt CIPRs in preference to ABPs.

3. Even if CIPRs are widely taken up, the current expectation is that typically only 25% of such products will be annuitised. If that is the case, then 75% of a typical CIPR will still require a decision by the retiree as to how much of their ABP component to draw down.

2.4 Assets outside super

The third "pillar" of retirement funding in Australia (alongside occupational superannuation and the age pension) is private saving. Some individuals will have saved some part of their wealth (aside from the family home) outside the superannuation environment. There can be a number of reasons for this, for example (but not limited to) the following:

[15] https://www.dss.gov.au/seniors-budget-measures/means-test-rules-for-pooled-lifetime-income-streams

- The assets are of a form that cannot be developed within a superannuation fund, for example an unincorporated private business.

- The individual reached their superannuation contribution limits and was unable to contribute the full amount of their available savings to superannuation.

- The individual preferred to invest outside superannuation because they did not wish to be bound by the regulatory constraints surrounding superannuation funds (e.g. constraints on borrowing).

- The individual wanted to keep some investments outside superannuation so that access to those funds prior to retirement date would not be an issue.

- The individual did not want to commit all their savings to a regime subject to so much policy change.

This paper assumes that the retiree holds no material financial assets outside the ABP. Likewise, this paper ignores the possibility of home equity conversion. Clearly however, in principle, assets outside superannuation are also available to fund retirement and should be considered by an individual when deciding his or her appropriate drawdown.

3 Relevant considerations

A rule of thumb in our context is a distillation of multiple factors into a simple decision rule relevant to the majority of a target group. Before compiling such a rule, it is necessary to consider the range of possible personal circumstances of individual retirees. In this Section, we consider twelve potentially relevant considerations and indicate the assumptions that we have made when developing improved drawdown rules in Section 6.

3.1 Demographic status

3.1.1 Single or Couple

Although it would be desirable to develop drawdown rules separately for couples as well as singles, we have considered only singles. In Section 6 we have applied dynamic programming techniques to compute the required parameters of the drawdown rules we have developed allowing for the complexity of the Australian means testing. It was necessary to avoid some of the additional dimensions of the problem for couples (such as age differences, the desired reversionary percentage to apply on first death, and the altered asset testing parameters after the first death) in order to keep the computations tractable.

Also, we note that ABPs attach to an individual, not a couple, and that the age-based statutory minimum drawdown rates apply to each individual.

3.1.2 Dependants

Owing to the age pension eligibility age being 65.5 and the additional complexity involved with the contrary assumption we have assumed that retirees have no dependants.

3.1.3 Gender

The greater expected longevity of females suggests that a slightly lower drawdown rate may be appropriate for a female of a given age than for a corresponding male, all other factors being equal. The fact that a female aged 65 is expected to live 22.05 years (using Australian Life Tables 2010-12, without adjustment for future mortality improvements) whereas a male is expected to live 19.22 years leads to an annuity value (the value of $1 payable continuously for life) at 3% real interest of 15.64 for females compared to 14.06 for males. Therefore, the sustainable rate of consumption for females is likely to be of the order of 10% lower for a female aged 65 than a male aged 65.

We note that no distinction is made according to gender under the statutory minimum ABP withdrawal rates.

Later in this paper when we develop a basis for deriving optimal drawdown rules, we compute optimal drawdown rates on specified assumptions separately for males in Appendix A.1 and for females in Appendix A.2. We developed this paper's suggested drawdown rules on the basis of male mortality but also tested the efficacy of those rules for females. The welfare gains of our rules for females (relative to drawing down at the statutory minimum rate) were similar to those for males.

3.2 Consumption pattern

We have assumed that the retiree's utility of consumption will be identical in all future years as in the current year, in real (AWE-indexed) terms. The rationale for defining "real" in AWE-indexed terms is firstly in order to keep the living standard of the retiree constant relative to the rest of the community, but secondly to keep the computations manageable given that the age pension is essentially linked to AWE[16].

We have therefore implicitly ignored any potential decrease in targeted real consumption with increasing age as the retiree moves from a more active phase of retirement to a more passive phase.

We have also ignored any large one-off spending items, and have assumed that any emergency fund is held outside the ABP.

We have not tested whether the income amounts produced by the various rules are "adequate" to meet retiree needs such as those identified by the ASFA Retirement Standards.[17]

3.3 Bequest motive

Given that the primary targets for our rules of thumb are full and part age pensioners, we believe that the first priority of the target group of retirees will be to support their future lifestyles in retirement rather than bequests (other than a bequest of the family home, see Section 3.12 below). This is consistent with the results of a National Seniors Australia (2018) survey[18] in which only about 25% of respondents stated that they believed that leaving an estate to their heirs was "very important". Moreover, of those respondents who believed that leaving an estate was very important, only 13% were prepared to minimise their own spending in order to preserve their estate. We therefore believe it is appropriate to ignore the bequest motive, at least in relation to the retiree's financial assets. We note that this approach is consistent with recent statements by senior politicians to the effect that the objective of

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[16] Social Security Guide Version 1.253 Released 20 March 2019, http://guides.dss.gov.au/guide-social-security-law/5/1/8/50.

[17] https://www.superannuation.asn.au/resources/retirement-standard.

[18] https://nationalseniors.com.au/uploads/07183036PAR_OnceBittenTwiceShy_Web_0.pdf

superannuation is to supplement or replace the age pension, not to facilitate tax-advantaged bequests to the next generation.

3.4 Other earned income

We have assumed that the retiree has no other current earned income.

3.5 Age Pension

We have assumed that the retiree is eligible to receive the full Age Pension, subject only to age and the means tests.

3.6 Other Assets

Whilst one might think that the optimal drawdown rate on ABP may be considered in isolation from any other assets held by the retiree, we show later in this paper that the optimal drawdown rate (as a percentage of the ABP) varies significantly according to the size of the ABP, primarily because of the operation of the Assets Test. But the same consideration applies to assets held outside superannuation: such assets (other than the family home) are not generally exempt from the Assets Test. Therefore, it has been necessary to assume, for the purposes of developing drawdown guidance, that there are no assets outside the ABP other than the principal residence of the retiree.

3.7 Age/longevity

We have explicitly taken age into account in considering the appropriate drawdown rate from a retiree's ABP. Other things being equal, in the absence of a bequest motive, a higher drawdown rate is appropriate for an older retiree with lower remaining life expectancy.

Whilst there are factors other than biological age that are conceivably relevant to life expectancy, these factors complicate the development of drawdown rules and we have made the simplifying assumption that all males and females are subject to the mortality rate relevant to their age as per the Australian Life Tables 2010-12. Adjustment for mortality improvement has been implemented by considering the ALT 25-year improvement factors adjusted for (Current Year - 2012 - 3) years. This is consistent with the mortality assumption used to generate the Australian Government Actuary's modelling results in Treasury's Retirement Income Covenant Position paper.[19].

[19] https://treasury.gov.au/sites/default/files/2019-03/c2018-t285219-position-paper-1.pdf

3.8 Asset allocation/investment strategy

Different asset allocations may reasonably be expected to produce different long-run mean returns from investment and also different volatilities. To the extent that riskier strategies (higher exposures to growth assets) are likely to lead to higher long-run mean investment returns, one might expect that optimal drawdown rates would be higher for those asset allocations.

Therefore, although we have derived the rules of thumb from a single set of "balanced" asset return and volatility assumptions (see Section 4.5 below), in Section 7 we have also considered the consequences for the expected total utility of consumption for those who adopt asset allocations that produce different return or volatility assumptions than our assumed default "balanced" asset allocation.

3.9 Economic/market conditions

In Section 6 of this paper where we develop drawdown rules, we have assumed that the drawdown should be a percentage of the ABP balance to maximise the utility of consumption. However, especially where the asset allocation adopted by the ABP account holder has a significant share of risky and volatile growth assets, the market value of the account may fluctuate from year to year. It would seem undesirable for consumption to be forced to respond entirely proportionally to relatively temporary movements in the market value of volatile assets.

Nevertheless, in this paper, we have not attempted to develop variants on our drawdown rules that would adjust the drawdown according to market conditions. Such an adjustment would require the user to make an additional calculation to obtain the adjusted drawdown rate, which was regarded as not being a sufficiently simple approach.

3.10 Tax

We have assumed in this paper that there is no tax on earnings within the ABP, nor is there any tax on payments from the ABP to the individual. For the target group (recipients of the full or partial age pension) these assumptions are likely to apply.

3.11 Regulatory minimum

Obviously, it is incumbent on the ABP pensioner to withdraw an amount at least equal to the statutory minimum, so any drawdown rule must always be subject to the statutory minimum percentage for the age of the ABP pensioner.

3.12 Principal residence

If the ABP account holder owns his or her own home, one question that may affect the appropriate drawdown rate is whether the ABP account holder regards their home as permanent and not to be touched under any circumstances, or whether the ABP holder is prepared to consider downsizing, either in the immediate future or at some future date.

Clearly this potential option cannot be absorbed into a drawdown rule without additional information about when the sale would occur and the likely proceeds. Therefore, any potential additional contribution to the ABP has been ignored in this paper.

3.13 Summary

In summary, for a range of reasons but mainly in order to keep the rule of thumb simple and the calculations tractable by limiting the number of variables, we have made the following assumptions for the remainder of this paper:

1. The ABP pensioner is single;

2. Calculations have been undertaken for males and females separately;

3. Large one-off consumption requirements and the holder's possible desire for an "emergency fund" have been ignored and consumption is to be considered in real AWE-adjusted terms not nominal terms, without any adjustment for possible reductions in consumption targets as the retiree moves from "active" to "passive" or "frail" status;

4. The pensioner attempts to optimise consumption and is treated as having no bequest motive;

5. All income drawn down is consumed;

6. The pensioner has no earned income from employment;

7. The ABP pensioner is eligible for the full Age Pension (subject only to the means tests);

8. The ABP pensioner has no material asset-testable assets outside their ABP;

9. The health and longevity prospects of the ABP holder are average for the relevant age and gender;

10. The ABP is invested in a "balanced" asset allocation;

11. No adjustment is to be made to smooth consumption according to the perceived under- or over-valuation of equities markets from time to time;

12. No tax is payable either on investment earnings within the ABP nor the pension payments from the ABP to the pensioner;

13. Any drawdown rule is of course subject to the regulatory minimum; and

14. The ABP holder either does not own a home or is not prepared to downsize in order to increase the balance of the ABP.

The authors recognise that the above constraints are quite restrictive but the constraints are needed to keep the calculations for Section 6 tractable. In any event, people of more substantial means will be better able to afford the comprehensive advice required to address the additional complexity of their circumstances.

4 Assessing rules using a utility framework

The utility framework has been used in academia for analysing complex problems involving consumption, savings and asset allocation for more than 50 years. Over the most recent couple of years, there have been more applications of the utility framework by industry practitioners. One example is the development of "The Members' Default Utility Function version 1" also known as MDUF v1[20], which was the outcome of 18 months of work by 14 respected academics and industry professionals.

4.1 Concepts and principles

Utility functions are mathematical functions that are mainly used to assess decisions under uncertainty and provide a basis for trade-offs between risk and rewards based on parameters determined by individual preferences. The use of a utility framework encourages the use of stochastic modelling, typically with simulations being performed for the lifetime of the member or pensioner.

The retirement problem is multi-dimensional as it involves making decisions to trade-off the level and stability of income received over the whole retirement period, and to trade-off the level of income and the access to capital/liquidity and potential bequest for beneficiaries. These decisions need to be made under consideration of risk factors such as investment risk and mortality risk. In this paper, we are focusing on assessing the income drawdown decisions by considering the trade-off between level and sustainability of income streams considering both investment and longevity risks for Australian retirees.

Utility functions can reduce the dimension of problems to something easier to measure and compare, by ranking alternative solutions according to their utility outcomes. They can then be used to compare alternative income drawdown rules. A higher utility outcome is preferred over a lower utility outcome. In theory, economically rational investors make decisions consistent with maximising the expected value of their utility.

4.2 Introduction to MDUF

The Member's Default Utility Function (MDUF) represents an attempt to quantify a sensible set of preferences for a superannuation fund trustee to assume on behalf of default fund members, on whom the trustees have limited information.

[20] Member's Default Utility Function (MDUF) is an open-architecture metric to assist the industry to design retirement outcome solutions. The related materials can be accessed through AIST website via http://www.aist.asn.au/policy/member%E2%80%99s-default-utility-function-(mduf).aspx and ASFA website via http://membersdefaultutilityfunction.com.au/

AIST and ASFA are custodians of all the relevant research including papers, models and presentation materials and they are free for all to access.

MDUF utilises constant relative risk aversion (CRRA) utility functions. For individuals who are CRRA, as their wealth increases, the percentage of their wealth invested in growth assets does not change. This means the dollar amount invested in growth assets will increase with wealth level. CRRA is the most widely used utility function, especially for the purpose of considering consumption over the life-cycle (Ameriks et al. 2011 and Lockwood 2014) as it is regarded as characterising relatively logical investor behaviour.

MDUF reflects the following preferences of a retiree that the working group believe are appropriate for a trustee to assume on behalf of a fund member:

1. The member is focussed on the future income stream not the retirement lump sum;

2. The member prefers a higher income stream;

3. The member prefers a less volatile income stream;

4. The member prefers not to outlive his or her retirement savings;

5. The member values residual benefits; and

6. The member is risk averse (the pain of an adverse outcome is greater than the joy of an equivalent positive outcome).

These preferences and the trade-off between them are then mathematically represented via a metric known as a utility function. MDUF is parameterised, establishing a sensible trade-off between the preferences. MDUF considers the risk faced by members and will heavily penalise poor retirement outcomes such as outliving one's financial assets, even if they have small possibilities of occurring.

The expected utility, U_0, is expressed in the form below:

$$U_0 = E_0\left[\sum_{t=0}^{T}\left\{{}_tp_x\,\frac{c_t^{1-\rho}}{1-\rho} + {}_{t-1|}q_x\left(\frac{\phi}{1-\phi}\right)^{\rho}\frac{b_t^{1-\rho}}{1-\rho}\right\}\right]$$

Where:

- T: time horizon
- c_t : income in year t
- b_t: residual account value at time t if the person dies between t-1 and t
- ${}_tp_x$: probability of being alive at age x+t conditional on being alive at age x
- ${}_{t-1|}q_x$: probability of dying between age x+t-1 and x+t conditional on being alive at age x

- ρ: level of risk aversion
- ϕ: strength of residual account ("bequest") motive

We have chosen to use a utility framework to assess drawdown rules instead of using other commonly used measures such as the replacement rate, probability of running out of money, shortfall risk and the funded ratio. This is because the utility framework implemented in MDUF captures the risk averse nature of retirees, in particular their aversion to income volatility, as well as the risk of outliving retirement savings (longevity risk) while the more commonly used measures fail to incorporate risk preferences.

4.3 MDUF parameterisation

4.3.1 Risk aversion

The risk aversion parameter ρ represents the consumption risk aversion level of an individual. A higher ρ represents a higher aversion to the risk of lower consumption. In most academic literature and relevant studies, the choice of the consumption risk aversion parameter ρ normally falls within the range of 1 to 8 with 5 being the most common choice. MDUF v1 specified the risk aversion parameter ρ to be 8, slightly more risk averse than the more typical choice of 5 in the academic literature.

Following Cocco et al. (2005), Gomes and Michaelides (2005), Pang and Warshawsky (2010), in this paper we use a base case risk aversion parameter ρ of 5 and then test the sensitivity of the results for ρ values of 2 and 8.

4.3.2 Residual benefit motive

As noted in Section 3.3, we have decided to focus our assessment on retirement income only and set the residual benefit motive strength parameter ϕ to be zero. The authors acknowledge that some retirees will have some motivation to leave residual benefits/bequests but, as discussed earlier, our target group comprises retirees on full or part age pension and we have decided to focus entirely on consumption outcomes rather than the residual estate.

4.3.3 Age pension entitlement

It is important to note that the version of MDUF that we used to score the alternative drawdown rules incorporated allowance for the age pension entitlements of the individual. Consumption that contributes to the "utility" core in MDUF may come equally from the amount of drawdown from the ABP, or from the age pension. The age pension entitlements were adjusted for both income from financial assets (using the deeming approach for the income test) and the assets test. It was assumed that the retiree had no personal exertion income and no assets outside the ABP for these computations.

4.4 Measures obtained from MDUF

Because we decided to focus on retirement income only, the expected utility can be simplified to the form below:

$$U_0 = E_0\left[\sum_{t=0}^{T} {}_tp_x \frac{c_t^{1-\rho}}{1-\rho}\right]$$

As CRRA utilities are defined on the negative domain and the utility values in their unadjusted form are hard to interpret, we define the following utility-related measures which we use for scoring drawdown rules in the paper:

- Risk-adjusted income ($ per annum): the constant level of real income that delivers an equivalent level of expected utility. Expected utility is calculated based on the formula above. This level of income is adjusted for income volatility as a result of investment risk and income shortfall risk. Risk-adjusted income also considers the probability that the retirees will receive each income stream payment.
- Welfare Gain of Lifetime Income ($): the additional initial wealth required for an inferior solution to achieve the same expected utility as a superior solution. Welfare loss is effectively a negative welfare gain.

4.5 Modelling assumptions

4.5.1 Investment Return

In this paper, we assume for the base case scenario, that retirees invest their retirement balance into a balanced investment option which is expected to provide 3.5% p.a. expected real return net of all fees[21] and tax if applicable and 7% annual return volatility[22]. The equity risk premium assumed is also broadly consistent with the ASX dividend yield after allowing for the impact of franking credits[23].

4.5.2 Mortality table

As discussed in Section 3.7, throughout this paper where mortality assumptions have been required, we have used the Australian Life Tables (ALT 2010-12) prepared by the Government Actuary adjusted for mortality improvement by including the ALT 25-year improvement factors for a period of (Current Year -

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[21] https://www.asx.com.au/documents/research/russell-asx-long-term-investing-report-2018.pdf

[22] Based on SuperRatings data in December 2018, the 10-year volatility for the median fund in the Balanced Fund group (60-76% growth asset) is around 6%. We added 1% on top of this to reflect the low-volatility environment in the post-GFC decade.

[23] https://www.actuaries.digital/2016/05/26/equity-risk-premium-survey-2015/

2012 - 3) years. This is consistent with Treasury's mortality assumption[24]. After the adjustments, we estimate a male retiree at age 65 has a residual life expectancy of 21.5 years and a female retiree at age 65 has a residual life expectancy of 23.9 years.

[24] Consistent with Treasury's mortality assumption used to generate Australian Government Actuary's modelling results in the Treasury's Retirement Income Covenant Position paper".

5 Some possible spending rules

5.1 "Spend the income"

One possible drawdown rule that might have an attraction to some pensioner members of SMSFs is to "spend the income but not the capital". The rationale for such a rule would be that no matter how long the pensioner survived, there would always be remaining assets since the capital has not been spent. However this approach suffers from the severe deficiency that the resulting spending would differ significantly depending on the underlying asset composition and periodic income. In addition, this approach is not readily available (without an ongoing advice relationship) to members of retail or industry funds where the usual approach to asset valuation is unitised without distinction between income and capital.

5.2 Bengen's 4% rule

Twenty-five years ago, the US financial adviser William Bengen proposed the "4% rule" (Bengen, 1994):

$C(t) = 0.04 * A(0) * CPI(t)/CPI(0)$

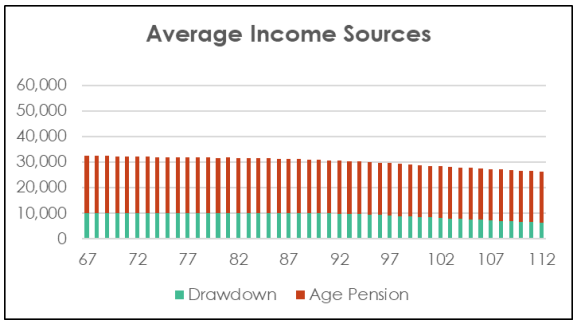
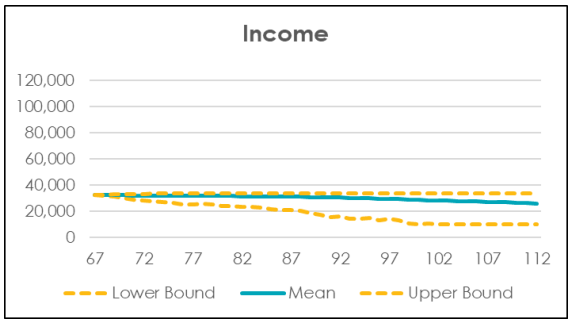
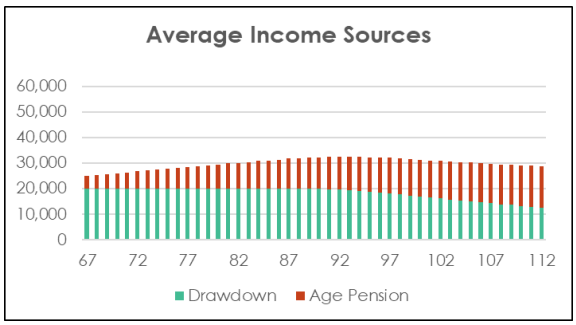
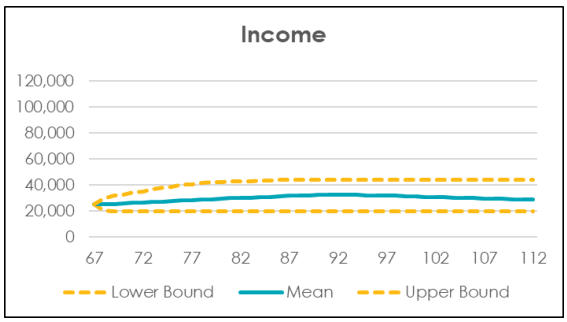
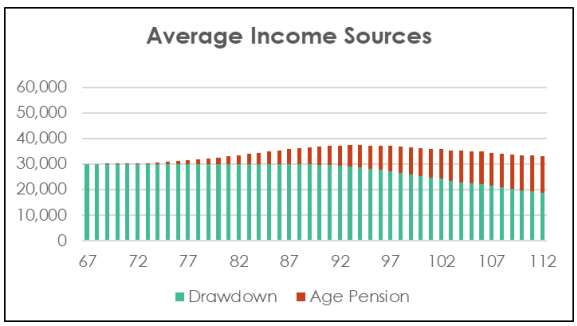
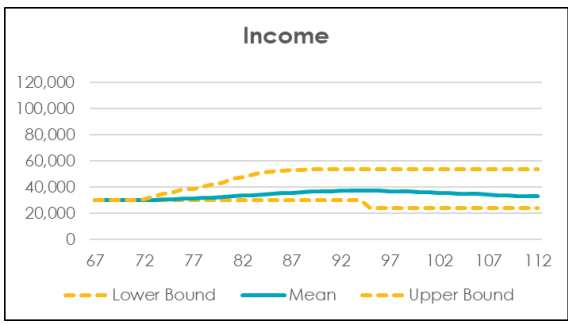
Where:

- C(t) = consumption in year t after retirement (t=0, 1, 2, …)
- A(0) = financial assets at the time of retirement (t=0)
- CPI(t) is the value of CPI at commencement of year t
- CPI(0) is the value of CPI at retirement date (i.e. the commencement of year 0)

In other words, the annual income to be drawn in the first year after retirement is 4% of assets at the date of retirement, and thereafter, the income to be drawn is increased with inflation.

Figure 1 shows the expected income (from the age pension as well as the indexed 4% drawdown) and the range of total income, for a single male homeowner with initial assets of $250,000, $500,000 and $750,000. The lower and upper bounds represent the bottom 5% and top 5% of the simulated outcomes.

Figure 1: Bengen's 4% rule (male, single, homeowner)

Expectation	Income Range
$250K	
	
$500K	
	
$750K	
	

Whilst Bengen's rule is simple and more widely known than any alternative rule, it has been criticised (Pfau, 2010; Drew and Walk, 2014) as not guaranteeing that assets will last for the lifespan of the retiree. Also, in the view of the authors, Bengen's rule seems intuitively unlikely to maximise total utility of consumption. In real-life scenarios, it seems likely that where investment returns have been favourable, total expected lifetime utility (measured by the Risk-Adjusted

Income and the Welfare Gain of Lifetime Income) could be improved by increasing consumption. Conversely, where investment returns have been unfavourable, it seems likely that total expected lifetime utility could be improved by reducing consumption to protect against the risk of the eventual expiry of funds and the damage to consumption utility that would result.

These intuitive expectations have been confirmed in Section 5.6 of this paper where we use utility-based methods to score Bengen's Rule and alternative rules.

5.3 NZ SOA 6% rule

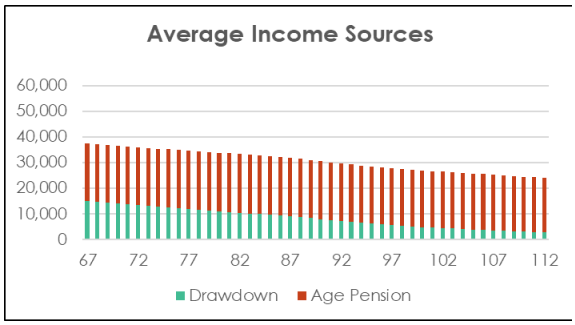
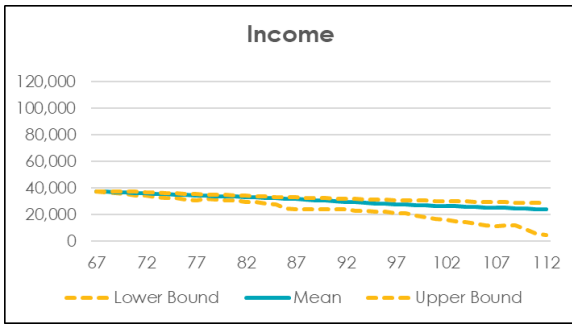
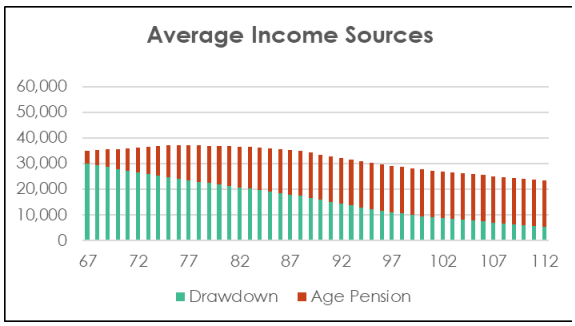
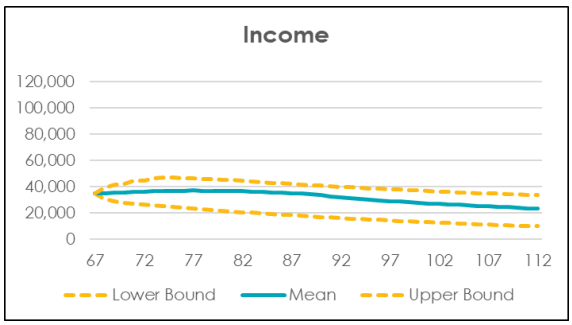
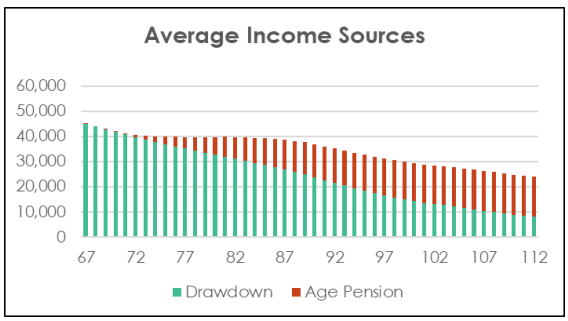
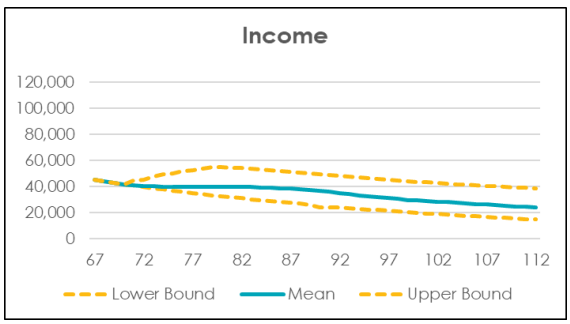
A Working Group of New Zealand actuaries (Retirement Income Interest Group of the New Zealand Society of Actuaries, 2017) assembled a list of "rules of thumb" for consideration by New Zealanders who might appreciate guidance on how to draw down on their KiwiSaver accounts, on retirement. KiwiSaver accounts are superannuation accounts, but they are not as tax-advantaged as in Australia and the average account balances are smaller than Australian superannuation average balances. The other significant difference between New Zealand and Australia is that the social security benefits are not means tested in New Zealand, which simplifies the guidance compared to Australia's means testing rules.

The Working Group suggested four "rules of thumb" of which Bengen's 4% rule was one. The second was the "6% rule" which is essentially very similar to the 4% rule except that the initial drawdown is 6% of assets at the date of retirement rather than 4%, and the dollar amount drawn does not increase with inflation.

Intuitively the 6% rule has less to recommend it than the 4% rule, because the real value of consumption may be expected to decline markedly over time. In addition, it suffers from the same weaknesses as identified above for the 4% rule: it is unlikely to be utility-maximising because often the actual sequence of investment returns will strongly suggest that optimal consumption should be higher or lower than the rule suggests.

The following charts show the expected income in real terms (from the age pension as well as the unindexed 6% drawdown) and the range of total income, for a single male homeowner with initial assets of $250,000, $500,000 and $750,000.

Figure 2: NZ SOA 6% rule (male, single, homeowner)

Expectation	Income Range
$250K	
	
$500K	
	
$750K	
	

As expected, the 6% rule did not score well when tested for utility in Section 5.6.

5.4 NZ SOA "life expectancy" rule[25]

The third rule of thumb proposed by the NZ SOA paper was aptly described as the "life expectancy" rule. Under this rule, recommended consumption in year t is calculated simply as

$C(t) = A(t)/e(x+t)$

where x is the retirement age, e(x) is the complete expectation of life at age x and C(t) and A(t) are the retiree's consumption and financial assets t years after the retirement date as defined in Section 5.2 above.

Of course, the life expectancy rule presupposes that the retiree has access to an appropriate life table. Whilst national life tables are readily available in both Australia and New Zealand, one possible technical criticism is that conventional "static" life tables do not allow for future mortality improvement, even though mortality has been showing very marked declines for more than a century.

Nevertheless, from an actuarial perspective, this seems more promising as a potential utility-maximising rule of thumb than the rules described in Sections 5.2 and 5.3 above. In particular it responds to the unexpected favourable or unfavourable asset returns, although it should be pointed out that to the extent that asset values are volatile (which will of course depend on the selected asset allocation), then the recommended consumption will be correspondingly volatile.

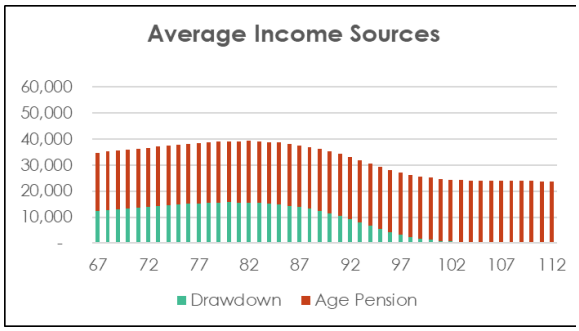
The following charts show the expected income (from the age pension as well as the drawdown according to the "life expectancy rule") and the range of total income, for a single male homeowner with initial assets of $250,000, $500,000 and $750,000.

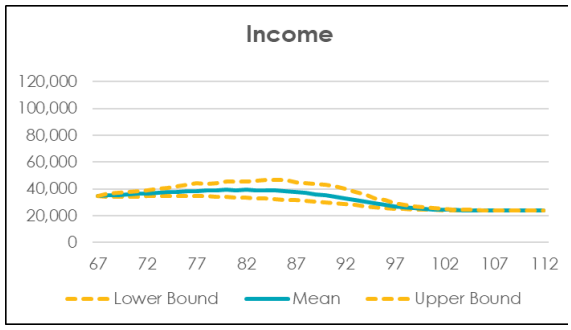
[25] A natural evolution and more actuarially pleasing version of this rule would be an "inverse of life annuity" rule. The authors developed such a rule as part of the drafting this paper but found optimal drawdowns a more effective base from which to develop their suggested rules of thumb.

Figure 3: NZ SOA Life Expectancy rule (male, single, homeowner)

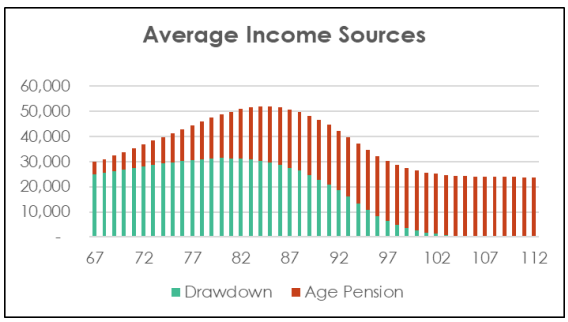
Expectation	Income Range

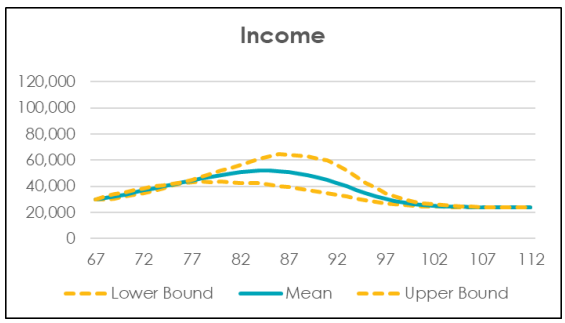
$250K



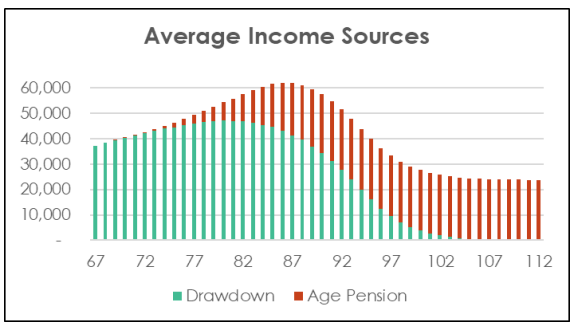


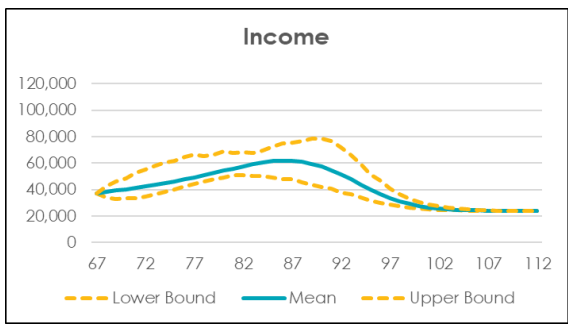
$500K





$750K





The Table below shows the drawdown factors used to prepare the above charts, based on the inverse of life expectancy.

Table 2: Inverse of life expectancy drawdown ratio (male, single, homeowner)

Age	Inverse of LE Drawdown %
67	5
68	5
69	5
70	6
71	6
72	6
73	6
74	7
75	7
76	8
77	8
78	8
79	9
80	9
81	10
82	11
83	11
84	12
85	13
86	14
87	15
88	17
89	18
90	20
91	21
92	23
93	25
94	26
95	28
96	29
97	30
98	31
99	32
100	34

5.5 Statutory minimum ABP drawdown rule

As noted earlier in this paper, although the minimum drawdown rule is the rule that appears to be followed by a high proportion of ABP pensioners, it was never intended as guidance to pensioners as the optimal rate at which to draw down on their pensions.

Another deficiency of the minimum drawdown rule is the income discontinuities introduced by quinquennial and decennial banding. From Table 1 we can see that there is an increase of 25% in the drawdown percentage as the ABP account holder turns 65; 20% on attaining age 75; 17% on attaining age 80; 28% on attaining age 85; 22% on attaining age 90; and 27% on attaining age 95. While in practice other factors, particularly investment performance, will influence the shape of the drawdown curve expressed in dollars, the step nature of the minimum drawdown percentages will generally serve to produce an undesirably serrated consumption curve.

The authors note that it appears that drawdown rates very nearly identical to the minimum statutory drawdown rates can be derived on the following basis:

Interest rate: 2.5% real interest

Mortality: Australian Life Tables 2010-12

Drawdown rate: $1/a_n$ where n is the lowest integer term such that ${}_np_x<0.05$ and $a_n=(1-(1.025)^{-n})/0.025$

In other words, the minimum drawdown rules appear to be consistent with a drawdown rate for a retiree aged x which is the inverse of a term certain annuity where the term certain is chosen to be long enough so that at least 95% of the population currently aged x will have passed away, with the calculation of the term certain annuity being evaluated at a 2.5% real interest rate.

The results of applying the approach described above are shown in Table 3. As may be seen from the Table, the calculated drawdown rates on the basis described above, rounded to the nearer 1%, exactly reproduce the male drawdown rates at most ages except for the older ages within some quinquennial bands, where the calculated rate may be marginally higher, and also reproduces the female drawdown rates at most ages except for the younger ages within some quinquennial bands, where the calculated drawdown rate is marginally lower than the statutory minimum. Averaged across both genders and across the age bands used for the statutory minimum rates and rounded to the nearest integer, the basis stated above reproduces exactly the percentages in the statutory minimum basis.

Table 3: Drawdown rates using term certain annuity at 2.5% real interest and 5% survival probability

Age	Males (%)	Females (%)	Statutory Minimum (%)
60	4	4	4
61	4	4	4
62	4	4	4
63	4	4	4
64	4	4	4
65	5	4	5
66	5	4	5
67	5	4	5
68	5	5	5
69	5	5	5
70	5	5	5
71	5	5	5
72	5	5	5
73	5	5	5
74	6	5	5
75	6	5	6
76	6	6	6
77	6	6	6
78	6	6	6
79	7	6	6
80	7	6	7
81	7	7	7
82	7	7	7
83	8	7	7
84	8	7	7
85	8	8	9
86	9	8	9
87	9	9	9
88	10	9	9
89	10	9	9
90	11	10	11
91	11	10	11
92	11	11	11
93	11	11	11
94	13	11	11
95	13	13	14
96	13	13	14
97	14	13	14
98	14	14	14
99	14	14	14

Having reverse-engineered an actuarial basis that is consistent with the statutory minimum drawdown rates, the authors observe that the minimum drawdown rule appears reasonable in relation to its objectives: both the interest assumption and the mortality assumption are somewhat (but not unreasonably) conservative, as seems appropriate for a legal minimum

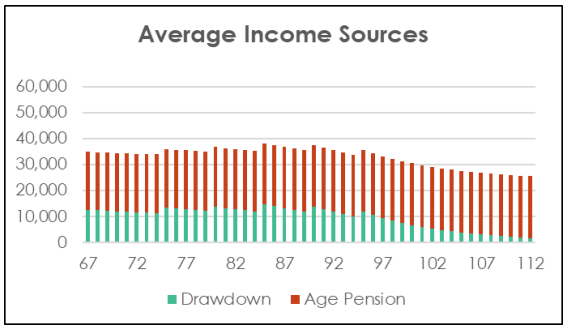
withdrawal rate. But a conservative drawdown rate is unlikely to be optimal in relation to the retiree's utility.

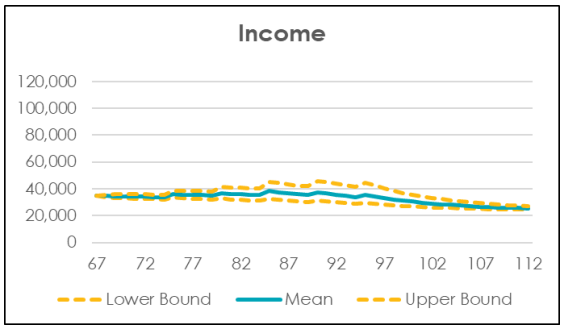
The charts in Figure 4 show the expected income (from the age pension as well as the drawdown according to the "statutory minimum drawdown rule") and the range of total income, for a single male homeowner with initial assets of $250,000, $500,000 and $750,000.

Figure 4: Statutory Minimum Drawdown (male, single, homeowner)

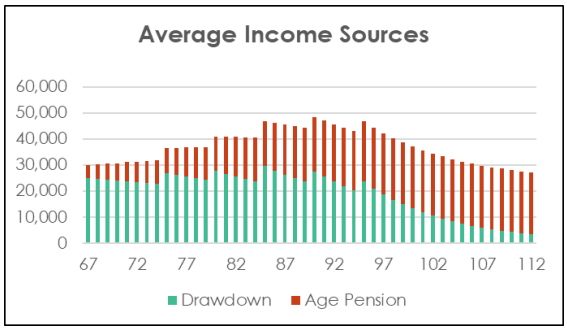
Expectation	Income Range

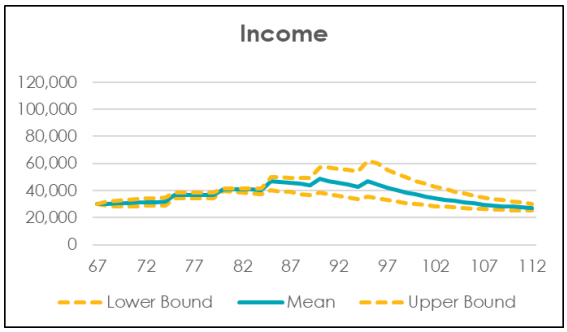
$250K



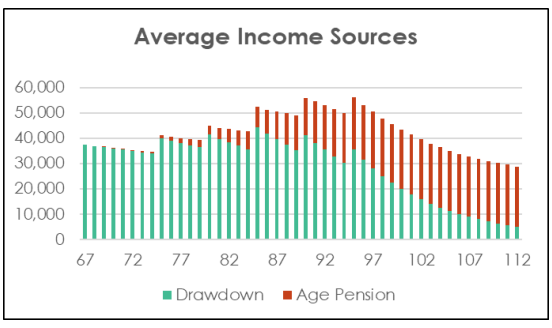


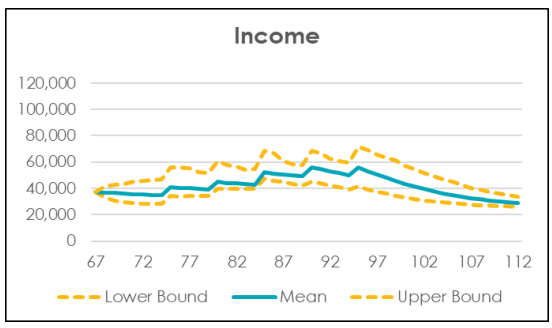
$500K





$750K





5.6 MDUF scoring of these rules

Table 4 below shows the MDUF "scoring" (see Section 4.4 above for the definitions of the "Risk-Adjusted Income" and the "Welfare Gain of Lifetime Income"). The final column in the table shows the actuarial present value (APV) of the Age Pension received if the retiree adopts each of the rules of thumb. It is calculated by discounting the Age Pension received at each future age at a real risk-free interest rate of 0% allowing for the probability of survival to each age.

We offer the following key observations on the results:

- Both Bengen's 4% rule and the NZ SOA 6% rule perform poorly in the Australian context, much worse than the Statutory Minimum drawdown rule. This is because both of the rules do not consider the investment experience over time and both entail a possibility that the income stream will expire completely.
- The NZ SOA Life Expectancy rule, however, performs much better. The fact that it automatically takes past investment performance into account (via the application of the rule to the actual assets held as at the beginning of each year) is a significant advantage relative to the 4% rule and the 6% rule. In fact the NZ SOA Life Expectancy rule is a better rule assessed through the MDUF metrics than the statutory minimum drawdown rule.
- Another important observation that can be inferred from Table 4 is just how penal the means tests are towards Australians with assets in excess of the lower Assets Test Threshold. The amount of age pension lost by an individual with $500,000 in testable assets rather than $250,000 by the time he or she reaches age pension eligibility age varies according to the drawdown approach chosen, and the amount of age pension foregone varies from $172,000 to $262,000. Effectively, therefore, the lifetime age pension foregone due to the additional $250,000 lies somewhere between 69% and 105% of the incremental $250,000, depending on the drawdown approach chosen. Similarly, the amount of age pension foregone by a single retiree who has accumulated $750,000 rather than $500,000 varies from $124,000 to $157,000, representing lifetime age pension foregone of between 50% and 63% of the incremental $250,000 in assets. The taper rate is clearly too high and the means tests need to be restructured to avoid both strong disincentives to save and strong incentives to indulge in a variety of pre-retirement financial planning practices which have little or no merit from a public policy perspective.

Table 4: MDUF assessments (ρ = 5, ϕ = 0, male, single, homeowner)

Retirement Strategy	Risk-adjusted Income ($000)	Welfare Gain of Lifetime Income ($000)	APV of Age Pension ($000)
$250K			
Statutory Minimum	35	-	461
Bengen's 4% Rule	27	-164	437
NZ SOA 6% Rule	28	-133	457
NZ SOA LE Rule	36	19	466
$500K			
Statutory Minimum	35	-	250
Bengen's 4% Rule	25	-203	175
NZ SOA 6% Rule	30	-96	258
NZ SOA LE Rule	38	67	294
$750K			
Statutory Minimum	39	-	93
Bengen's 4% Rule	31	-150	51
NZ SOA 6% Rule	36	-55	113
NZ SOA LE Rule	44	106	155

6 Better drawdown rules

Noting that some of the various simple rules of thumb proposed in Sections 5.1 to 5.4 score poorly on the MDUF appraisal in Section 5.6, two reasonable questions to ask are 1) whether it is possible to produce drawdown rules that will produce better retirement outcomes based on the utility measures; and 2) whether it is possible to produce a better rule of thumb, even if it is acknowledged that the rule of thumb will not be optimal in all circumstances. In this Section, we address these questions by starting from the optimal drawdown rates and then simplify them while not compromising too much on retirement outcomes.

6.1 Optimal drawdown

Butt, Khemka and Strickland (2018) present a dynamic programming approach to solving simultaneously for the optimal asset allocation and drawdown based on the consumption utility framework described in Section 4 of this paper. While we enthusiastically endorse this approach, in practice it is more likely that many retirees will approach the issues of asset allocation and drawdown separately and may well resolve to adopt a particular asset allocation independently of their drawdown decisions. With the focus of this paper being the drawdown decision, we pre-determined the asset allocation (balanced fund) and adopted the same dynamic programming methodology to derive the optimal drawdown decision. Appendix A.1 shows the optimal drawdown rates for each age and asset balance based on ρ (the risk aversion parameter) being 5.

The charts in Figure 5 show the projected income resulting from applying the optimal drawdown approach for risk aversion parameter ρ=5.

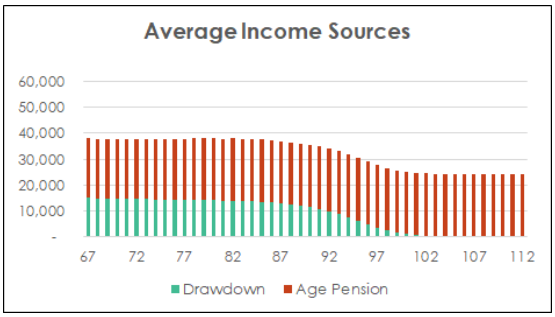
Whilst the optimal drawdown is by definition optimised (with $\boldsymbol{\rho}$=5, no bequest motive and balanced asset allocation), finding the optimal drawdown involves looking up the large two-dimensional matrix of ages and asset balances in Appendix A.1.

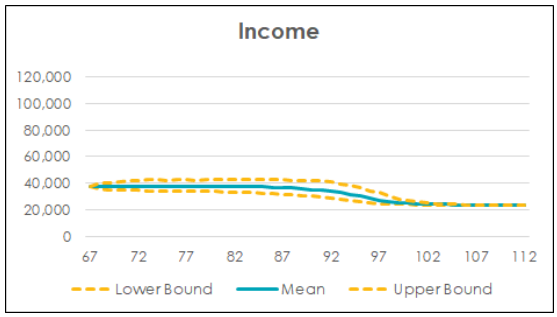
The next three Sections represent alternative attempts to adapt the pattern of percentages in the optimal drawdown table in Appendix A.1 to straightforward calculation rules that might be more digestible to retirees with different levels of financial sophistication.

Figure 5: Optimal drawdown rules ($\rho = 5$, $\phi = 0$, male, single, homeowner)

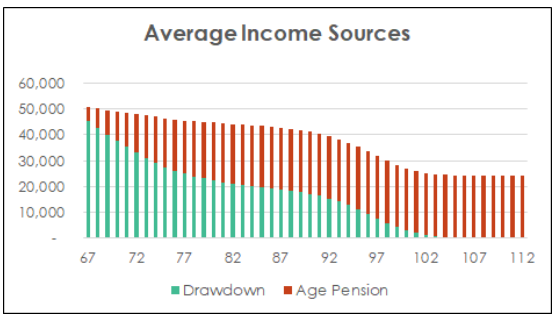
Expectation	Income Range

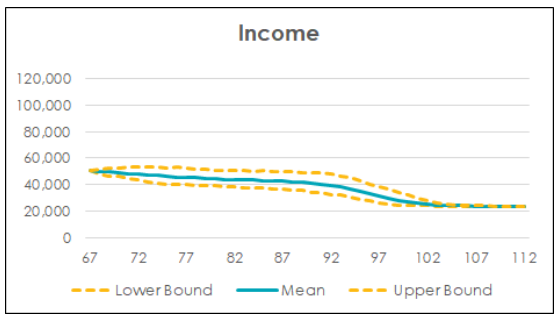
$250K



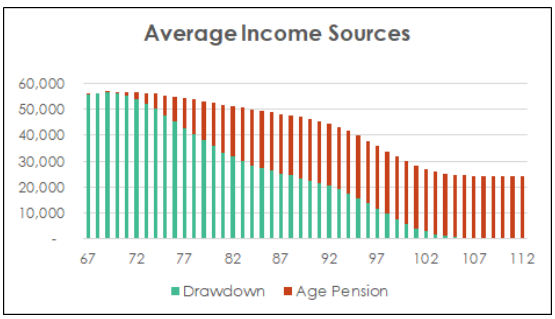


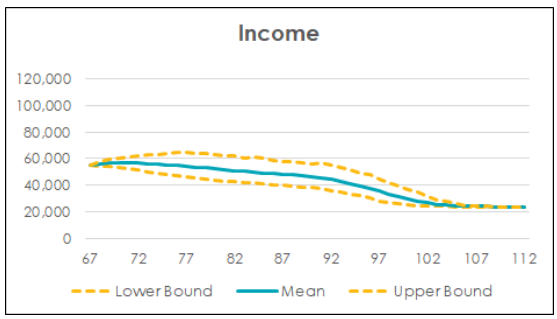
$500K





$750K





6.2 A rule of thumb

The simplest of our rules may be expressed as follows:

- Baseline drawdown rate % = the first digit of your age
- Add 2% to the baseline drawdown rate if your account balance is between $250K and $500K (exclusive)
- Subject to the statutory minimum drawdown rule

By way of illustration, the recommended baseline drawdown rate for someone in their 60s would be 6%. If the person has an ABP balance between $250K and $500K, then the recommended drawdown rate would be 8%.

The aim of the rule of thumb is to provide simple and easy to follow guidance that does not require retirees to do additional research. The account balance threshold $250K and $500K are derived based on the current Age Pension asset test thresholds, and thus subject to indexation with AWOTE. For simplicity, the thresholds are indexed and rounded to the nearest $50K, which means the account balance thresholds for a retiree to apply the rule of thumb would only need to be updated every few years with minimal complexity. The minimum statutory drawdown rate would start to take effect for people aged 85 or more, as the guidance would be to spend 8% of assets, whereas the statutory minimum from age 85 is 9%.[26]

Appendix B.1 shows the derived results for the rule of thumb in an expanded table. Figure 6 below shows the projected income for retirees under the rule of thumb with different initial asset levels at retirement date.

[26] We note that the rule of thumb, like all other simple rules in this paper, is subject to discontinuities at decennial ages as the retiree's age increments from age 59 to 60, 69 to 70 and 79 to 80. A closely related alternative to rule A would be rule A':

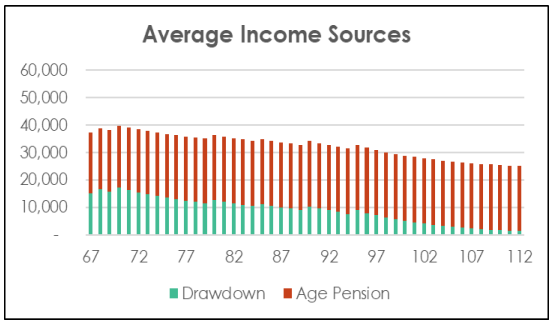
- Baseline drawdown rate % = [(x-5)/10]%
- Drawdown rate % for people with balance between $250K and $500K (exclusive) = [(x-5)/10+2]%
- Subject to the statutory minimum drawdown rule

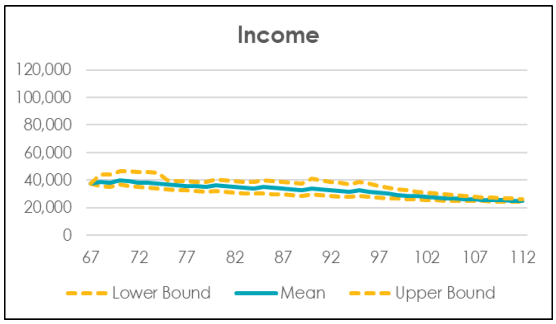
where x represents age. We have not separately projected income under this modified rule but, given the two rules are so closely related, we are confident that the shape of the modified projections would be much the same as the unmodified projections, but without the discontinuities at ages 60, 70 and 80.

Figure 6: Rule of Thumb (male, single, homeowner)

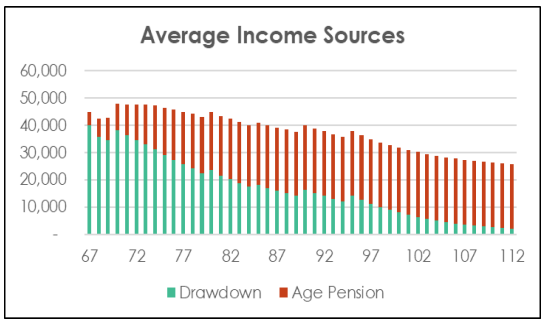
Expectation	Income Range

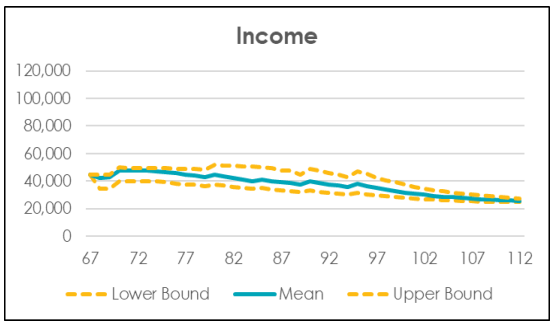
$250K



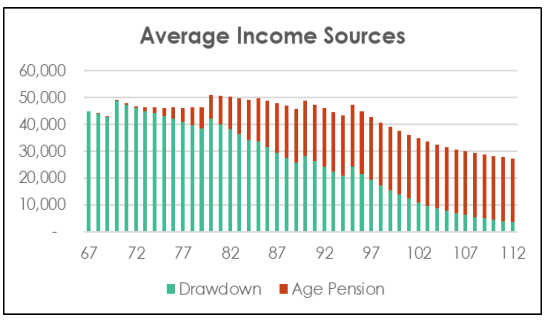


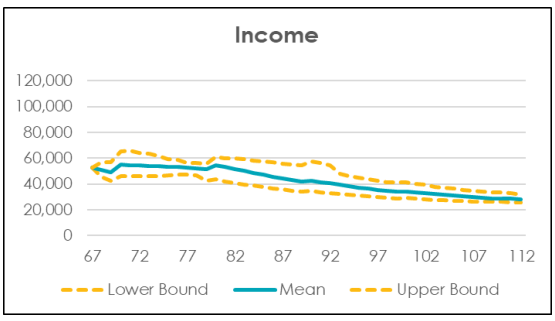
$500K





$750K





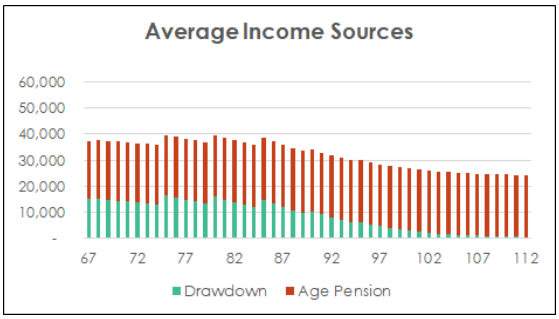
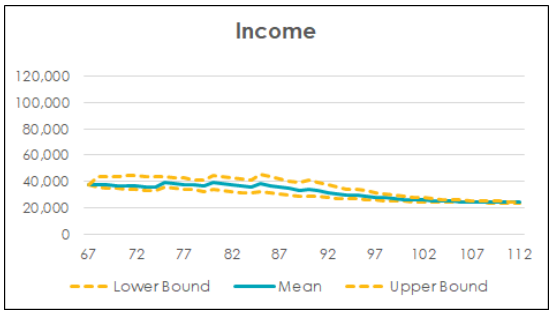
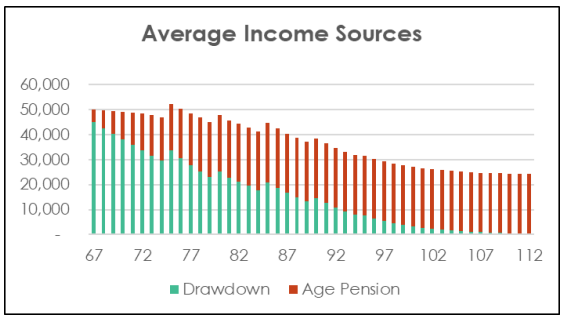
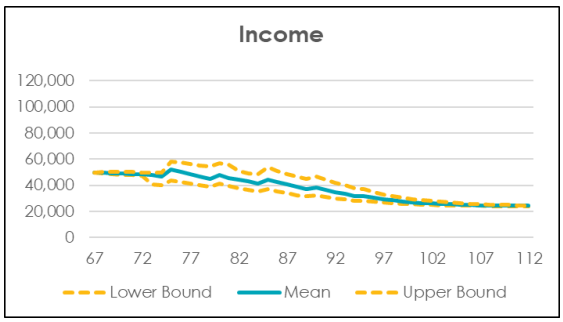
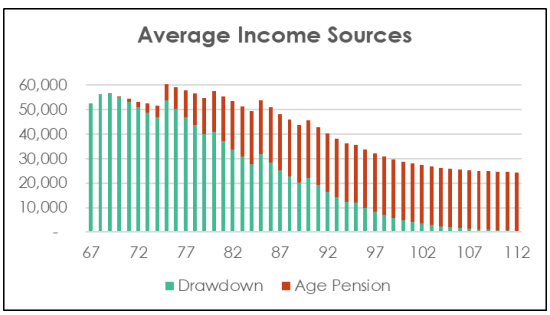
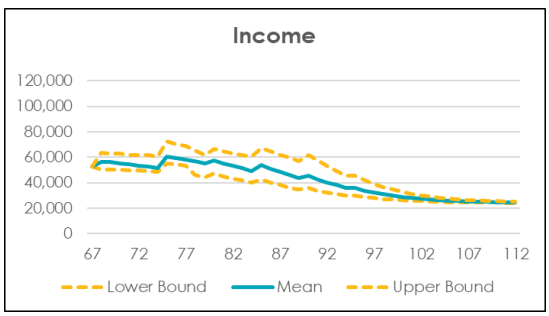
6.3 A rule for financially sophisticated retirees

Our second rule (shown in Table 5) uses a modestly sized two-way lookup table to determine the drawdown rate. This rule might be used by retirees who are more financially engaged and who are willing to vary their chosen percentage drawdown rate a little more frequently according to age and asset balance, as per Table 5 below. Ages are banded quinquennially, but no distinction is made between ages below 75, or after age 95. We derived this rule by grouping and averaging the optimal drawdown ratios based on age and balance ($\rho = 5$, $\phi = 0$, male, single, homeowner) as shown in Appendix A.1. Appendix B.2 shows the derived results of this rule in an expanded table. Figure 7 shows the projected income using this drawdown rule.

Table 5: Sophisticated retiree rule derived from optimal drawdown ratios ($\rho = 5$, $\phi = 0$, male, single, homeowner)

(%)	<=74	75 - 79	80 - 84	85 - 89	90 - 94	>= 95
<= 260K	6	8	10	13	15	17
261K - 700K	8	10	11	13	15	17
701K - 1M	7	8	9	11	13	15
> 1M	6	7	8	10	12	15

Figure 7: Rule for financially sophisticated retirees (male, single, homeowner)

Expectation	Income Range
$250K	
	
$500K	
	
$750K	
	

6.4 A rule for financial advisers

The third rule that we have derived directly from the optimal drawdown analysis is represented below in Table 6. Here age progresses in single years, rather than being grouped quinquennially or decennially, and assets are grouped in $100K bands rather than the $10K bands in the original dynamic programming analysis in Appendix A.1. Appendix B.3 shows the expanded table corresponding to this rule. Figure 8 shows the projected income from adopting this "financial adviser" rule.

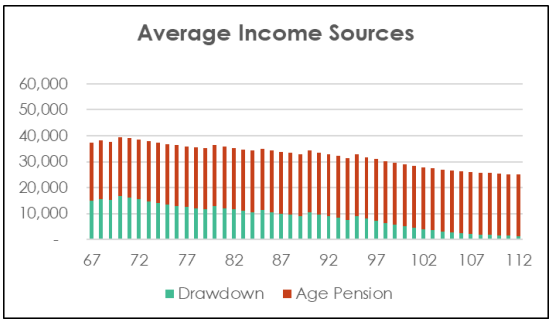
Table 6: Financial adviser rule derived from optimal drawdown ratios (ρ = 5, ϕ = 0, male, single, homeowner)

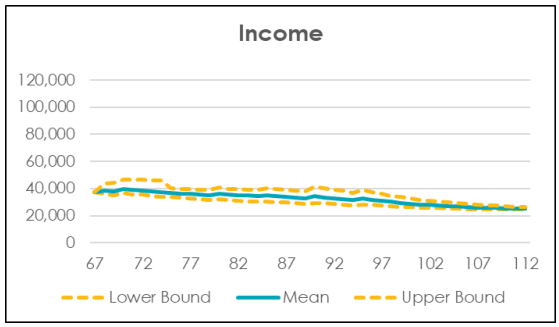
	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	>= 95
>700K	6	6	6	7	7	7	7	7	7	7	8	8	8	8	9	9	9	9	10	10	10	11	11	12	12	13	13	14	14
601 - 700K	8	9	9	9	9	9	9	9	10	10	10	10	10	11	11	11	12	12	12	13	13	13	14	14	15	15	16	16	17
501 - 600K	9	9	9	10	10	10	10	10	10	11	11	11	11	11	12	12	12	13	13	13	14	14	15	15	16	16	17	17	18
401 - 500K	9	9	9	9	9	9	10	10	10	10	10	11	11	11	11	12	12	12	13	13	14	14	15	15	16	16	17	17	18
301 - 400K	8	8	8	8	8	8	8	9	9	9	9	10	10	10	10	11	11	12	12	12	13	13	14	14	15	16	16	17	17
201 - 300K	6	6	6	6	7	7	7	7	7	8	8	8	9	9	9	10	10	11	11	12	12	13	13	14	15	15	16	16	17
<= 200K	5	5	5	6	6	6	7	7	7	8	8	9	10	10	11	11	12	13	14	15	16	17	18	19	20	21	22	24	24

Figure 8: Rule for financial advisers (male, single, homeowner)

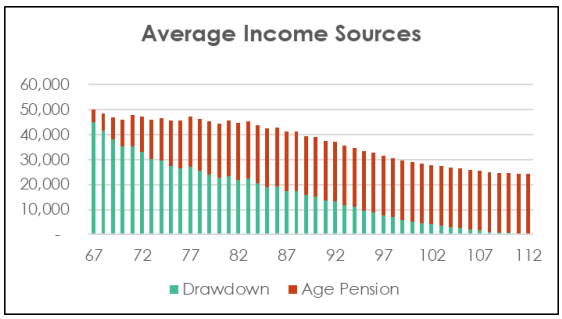
Expectation	Income Range

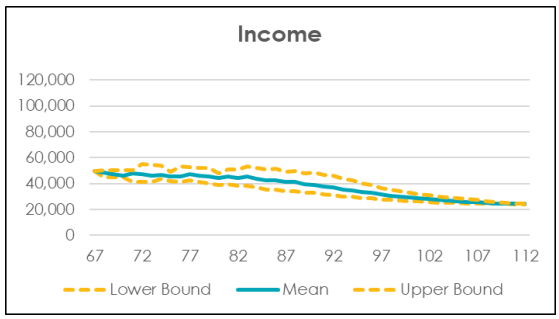
$250K



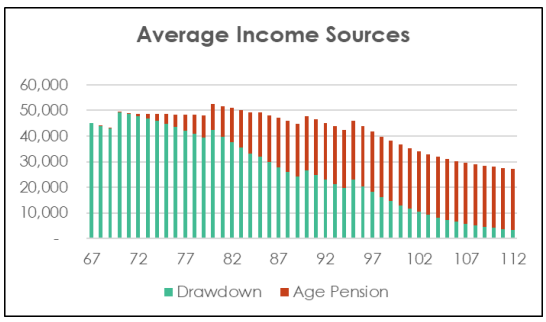


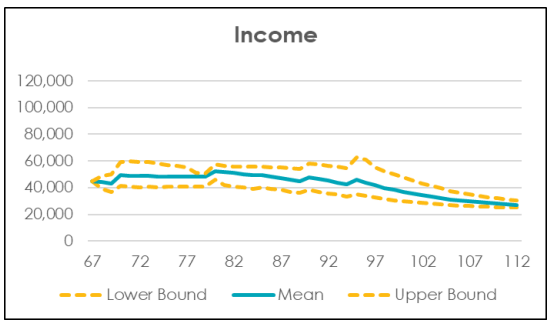
$500K





$750K





6.5 MDUF assessment

Table 7 presents the relative MDUF "scoring" of the three alternative rules developed.

Obviously, one would expect the more granular rules (the "financial adviser" rule is more granular than the "financially sophisticated retiree" rule, which in turn is more granular than the rule of thumb) to yield better MDUF scores given that the three rules were all derived from the same dynamic programming analytical starting point. Whilst Table 7 confirms that expectation, it is worth noting that the differences between the MDUF assessments of the rules of thumb are small for the $250K asset level, larger for the $500K asset level, and largest for the $750K asset level. This is attributable to the fact that the ABP drawdown is a much larger component of the expected income of someone with $750K in assets than someone with $250K, whose largest income source will be the (very stable) age pension. As may be seen, all rules always perform better than the statutory minimum, but the rule of thumb does not perform quite as well as the other rules for the highest balance level ($750K).

Table 7: MDUF assessments ($\rho = 5$, $\phi = 0$, male, single, homeowner)

Retirement Strategy	Risk-adjusted Income ($000)	Welfare Gain of lifetime income ($000)	APV of Age Pension ($000)
$250K			
Statutory Minimum	35	-	464
Rule of Thumb	35	7	472
Sophisticated Retiree Rule	36	16	471
Financial Adviser Rule	36	16	472
Optimal Drawdown	36	22	471
$500K			
Statutory Minimum	35	-	252
Rule of Thumb	42	134	352
Sophisticated Retiree Rule	42	151	354
Financial Adviser Rule	43	156	361
Optimal Drawdown	43	165	368
$750K			
Statutory Minimum	39	-	94
Rule of Thumb	45	134	163
Sophisticated Retiree Rule	48	194	214
Financial Adviser Rule	49	203	237
Optimal Drawdown	49	214	237

Appendix A.2 demonstrates that the optimal drawdown rule derived based on female mortality is not dramatically different from the one derived based on male mortality. As a result, we assessed how the derived rules would perform by applying them to the female case and the results are shown in Table 8. As expected, all three rules of thumbs provide better outcomes compared to the statutory minimum for all three ABP account balances. Implementing these rules rather than the statutory minimum also provides considerable welfare gains that are comparable to the maximum welfare gains that can be achieved through implementing the optimal drawdown rules designed based on female mortality.

Table 8: MDUF assessments ($\rho = 5$, $\phi = 0$, female, single, homeowner)

Retirement Strategy	Risk-adjusted Income ($000)	Welfare Gain of lifetime income ($000)	APV of Age Pension ($000)
$250K			
Statutory Minimum	35	-	519
Rule of Thumb	35	2	527
Sophisticated Retiree Rule	35	10	526
Financial Adviser Rule	35	10	527
Optimal Drawdown	36	16	527
$500K			
Statutory Minimum	35	-	295
Rule of Thumb	41	129	403
Sophisticated Retiree Rule	42	142	406
Financial Adviser Rule	42	149	414
Optimal Drawdown	42	156	420
$750K			
Statutory Minimum	39	-	122
Rule of Thumb	45	137	202
Sophisticated Retiree Rule	47	184	259
Financial Adviser Rule	47	192	284
Optimal Drawdown	48	202	282

7 Sensitivity Analysis

In this Section of our paper we consider how our proposed rules would impact on retirees for whom, for some reason, our assumptions do not apply.

7.1 Risk aversion levels

We start by considering the consumption risk aversion parameter **ρ**. Tables 9 and 10 consider the situation where the retiree is less risk averse with **ρ** of 2 (Table 9) and where the retiree is more risk averse with **ρ** of 8 (Table 10), but the individual nevertheless follows the drawdown guidance rules developed from assuming a **ρ** value of 5.

As shown in Table 9 and 10, although the three rules in Sections 6 were derived based on a risk aversion parameter **ρ** of 5, they are still always preferable to the statutory minimum drawdown rule if the retiree is actually more risk averse or less risk averse than the base case. The more granular "financially sophisticated retiree" and "financial adviser" rules produce welfare gains approaching the maximum welfare gain from implementing the optimal drawdown strategies.

Table 9: MDUF assessments (ρ = 2, ϕ = 0, male, single, homeowner)

Retirement Strategy	Risk-adjusted Income ($000)	Welfare Gain of lifetime income ($000)	APV of Age Pension ($000)
$250K			
Statutory Minimum	35	-	464
Rule of Thumb	36	12	472
Sophisticated Retiree Rule	36	23	471
Financial Adviser Rule	36	22	472
Optimal Drawdown	37	32	469
$500K			
Statutory Minimum	36	-	252
Rule of Thumb	43	130	352
Sophisticated Retiree Rule	44	150	354
Financial Adviser Rule	44	153	361
Optimal Drawdown	45	178	367
$750K			
Statutory Minimum	40	-	94
Rule of Thumb	46	118	163
Sophisticated Retiree Rule	50	196	214
Financial Adviser Rule	51	210	237
Optimal Drawdown	53	245	231

Table 10: MDUF assessments (ρ = 8, ϕ = 0, male, single, homeowner)

Retirement Strategy	Risk-adjusted Income ($000)	Welfare Gain of lifetime income ($000)	APV of Age Pension ($000)
$250K			
Statutory Minimum	35	-	464
Rule of Thumb	35	2	472
Sophisticated Retiree Rule	35	8	471
Financial Adviser Rule	35	8	472
Optimal Drawdown	35	15	469
$500K			
Statutory Minimum	34	-	252
Rule of Thumb	40	130	352
Sophisticated Retiree Rule	41	138	354
Financial Adviser Rule	41	147	361
Optimal Drawdown	42	158	367
$750K			
Statutory Minimum	37	-	94
Rule of Thumb	44	143	163
Sophisticated Retiree Rule	45	170	214
Financial Adviser Rule	46	178	237
Optimal Drawdown	47	199	231

7.2 Investment return and volatility

We next consider the investment return parameters. Table 11 considers the situation where the expected real return of the ABP is 2.5% and Table 12 considers the situation where the volatility of the ABP is 10%. Although the MDUF scores are different from the base case, the relativity between the five drawdown rules still holds the same in both situations.

Table 11: MDUF assessments (mean = 2.5%, vol = 7.0%, male, single, homeowner)

Retirement Strategy	Risk-adjusted Income ($000)	Welfare Gain of lifetime income ($000)	APV of Age Pension ($000)
$250K			
Statutory Minimum	34	-	469
Rule of Thumb	34	4	474
Sophisticated Retiree Rule	35	13	473
Financial Adviser Rule	34	12	474
Optimal Drawdown	35	19	473
$500K			
Statutory Minimum	35	-	291
Rule of Thumb	40	104	370
Sophisticated Retiree Rule	41	118	370
Financial Adviser Rule	41	122	376
Optimal Drawdown	41	131	378
$750K			
Statutory Minimum	37	-	133
Rule of Thumb	44	127	133
Sophisticated Retiree Rule	46	170	242
Financial Adviser Rule	46	177	262
Optimal Drawdown	47	189	257

Table 12: MDUF assessments (mean = 3.5%, vol = 10%, male, single, homeowner)

Retirement Strategy	Risk-adjusted Income ($000)	Welfare Gain of lifetime income ($000)	APV of Age Pension ($000)
$250K			
Statutory Minimum	35	-	457
Rule of Thumb	35	9	470
Sophisticated Retiree Rule	35	18	469
Financial Adviser Rule	35	18	468
Optimal Drawdown	36	24	469
$500K			
Statutory Minimum	35	-	246
Rule of Thumb	41	119	338
Sophisticated Retiree Rule	42	139	345
Financial Adviser Rule	42	144	353
Optimal Drawdown	43	152	359
$750K			
Statutory Minimum	39	-	105
Rule of Thumb	45	128	168
Sophisticated Retiree Rule	48	181	212
Financial Adviser Rule	48	187	233
Optimal Drawdown	49	199	231

8 Discussion

In this paper, we have used a dynamic programming technique to derive the optimal drawdown rates by age and account balance and then derived drawdown rules including a simple rule of thumb based on the optimal outcomes. The dynamic programming technique we used is consistent with that described in Butt, Khemka and Strickland (2017) and our optimal drawdown ratios results also reconcile. The key challenge of this paper is to identify robust rules that cope with the means testing of the age pension, different investment environments and different pensioner risk aversion levels.

However, before we proceed to summarise our key conclusions in the next Section, we should comment on some of the methodological aspects that underlie our paper, on some of our assumptions, and on further work that the authors consider would be highly desirable.

8.1 Utility curve methodology

We have built our analysis on a consumption-utility framework and we believe that this framework provides a strong conceptual basis for our analysis. In the process, though, we have skipped relatively lightly over issues such as:

- how to assign a value of ρ to an individual investor;
- whether the value of ρ for an individual is typically stable enough over time to yield a robust conclusion;
- the implications of including a residual benefit motive; and
- the implications of non-CRRA utility frameworks such as prospect theory (Kahneman and Tversky 1979), habit persistence (Constantinides, 1990), and consumption utility relative to a consumption floor (Kingston and Thorp, 2005 and Ganegoda and Bateman, 2008).

Whilst these issues may have received some study in the literature, we have not tried to evaluate their overall appropriateness of the implicit assumption that individuals have CRRA utility or whether the utility curve approach is practicable in practice.

8.2 Assumptions

The dynamic programming calculations we undertook in Section 6 to help identify suitable drawdown rules are complex, and it has been necessary to make some simplifying assumptions in order to keep the task manageable. For example, we have assumed that the consumption inflation is linked to AWE instead of CPI - this assumption kept the calculations manageable because we allowed for the age pension entitlements that are effectively linked to AWE. If we had not been constrained to keep the calculations manageably simple, we may well have not made this assumption. For example, we might have linked inflation to CPI rather than AWE, or we might have introduced utility curves which allowed for reduction in real spending over time as the retiree ages, become more frail and less willing or able to engage in extensive travel away from home.

In addition, it has been necessary to make many assumptions about the personal circumstances of the retiree. We acknowledge that these assumptions are somewhat restrictive and that there may be many retirees to whom our assumptions would not apply. An obvious example is that we have only developed rules for the case of single retirees. Ideally, we would have tested the "personal circumstances" assumptions we have made against census or similar data to gauge their range of applicability.

The mathematics would be more difficult for couples, because there are additional variables involved, but clearly restricting our work to be specifically relevant only to singles is a deficiency.

We have also assumed that income generated by ABP is entirely consumed.

Ideally, developments in financial software at some future date might mean that individuals can input their own personal details, and the dynamic programming techniques will produce appropriate advice for the individual allowing for all the relevant circumstances.

8.3 Further work

Some of the further work that the authors believe should be addressed in future research is listed below.

1. Research on the task of assigning consumption risk aversion parameters to individual retirees and to evaluate the reliability of the assessed ρ values.
2. Work developing drawdown rules (including a rule of thumb) for couples.
3. Work applying utility-based analysis to evaluating the desirability of alternative product mixes that a retiree might wish to consider (for example, 25% immediate annuity plus 75% ABP).
4. Work on software techniques to enable a wide range of retiree circumstances to be investigated and allowed for using sophisticated optimising approaches.
5. Work on a wider range of sensitivity calculations.

In addition, spending rules of the Yale-endowment type have intuitive appeal to several of the authors and they could be investigated for applicability in the retirement context.[27]

[27] See, for example, https://www.endowmentinvestor.org/my_weblog/2014/03/the-yale-spending-rule.html.

9 Conclusions

Notwithstanding some of the methodological issues highlighted in Section 8 above, the authors believe that the paper has demonstrated the following conclusions.

1. Although it is not possible to find drawdown rules that are simple and optimal for everyone, on the assumptions stated in Sections 3 and 4 of our paper, the rules we have developed including the "rule of thumb" in Section 6 all provide better retirement outcomes for retirees than drawing down at the statutory minimum rate.
2. All the rules we have developed respond significantly to the age pension means testing parameters, with recommended drawdown rates being materially higher within the asset testing range (particularly towards the top of the asset testing range and for older retirees). However, most Australians seem to draw down on their ABPs at (or close to) the minimum statutory rate. There could be several explanations for this phenomenon, for example: (a) that many do not fully grasp the implications of the means tests for current and future consumption; (b) some have a bequest motive; or (c) retirees value factors that are not captured by the consumption utility framework such as maintaining contingency amounts for health costs or aged care, and a preference for flexibility; or (d) the "nudge" effect of the statutory minimum dominates other issues.
3. All the rules we have proposed involve discontinuities around age and asset balances. Whilst on any view, a rule of thumb should not involve complex interpolation formulae, equally it does not seem to be ideal that the practical solutions to the question "how much can I drawdown" imply sizeable discontinuities from year to year.
4. Another potential source of discontinuity in the drawdown guidance is movement in the market values of assets. Whilst discontinuity issues may difficult to address via a rule of thumb, if retirees are able to use a tool or application to help guide their drawdown, it would be highly desirable for the tool or application to incorporate smoothing mechanisms. Such mechanisms may not significantly increase the MDUF assessed scores but should increase them modestly and would seem appropriate in the context of the tool or application.
5. The complexity of individuals' personal circumstances emphasises the need for financial advice, yet few Australians are willing to pay for such advice. Excellent techniques for computing optimal spending for individuals are available but these are complex and currently not available to individuals. It would seem desirable for individuals to have access to tools into which they could input their own financial and personal data. The ultimate objective would be to enable good advice to be provided at a very modest cost.
6. Above all, we believe that our paper has identified that the most widely used reference, the statutory minimum drawdown rules, is generally too low to yield anything close to optimal utility. Individuals would be better advised to spend more, especially in the younger years of retirement. This finding is consistent

with previous Australian studies (Spicer, Stavrunova and Thorp, 2016; Asher, Meyricke, Thorp and Wu, 2017).

7. Whilst we have regarded it as preferable in the context of a rule of thumb to keep the rule simple by using whole percentages of assets, and typically broad age bands, if the guidance were being produced automatically by a suitable "app", the guidance could be structured to follow more granular age and asset categories which would be likely to lead to smoother spending and slightly increased MDUF scores.

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Appendix A Optimal drawdown analysis

A.1 Optimal drawdown based on balance and age ($\rho = 5$, $\phi = 0$, male, single, homeowner, mean = 3.5%, vol = 7%)

	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100	101	102	103	104	105	106	107	108
1,500,000	5	5	5	6	6	6	6	6	6	6	7	7	7	7	8	8	8	8	9	9	9	10	10	11	11	11	12	12	14	14	14	15	15	16	17	19	20	22	26	30	38	53
1,480,000	5	5	6	6	6	6	6	6	6	7	7	7	7	7	8	8	8	8	9	9	9	10	10	11	11	11	12	12	14	14	14	15	15	16	17	19	20	22	26	30	38	53
1,460,000	5	5	6	6	6	6	6	6	6	7	7	7	7	7	8	8	8	8	9	9	9	10	10	11	11	11	12	12	14	14	14	15	15	16	17	19	20	22	26	30	38	53
1,440,000	5	5	6	6	6	6	6	6	6	7	7	7	7	7	8	8	8	8	9	9	9	10	10	11	11	12	12	13	14	14	14	15	15	16	17	19	20	22	26	30	38	53
1,420,000	5	5	6	6	6	6	6	6	6	7	7	7	7	7	8	8	8	8	9	9	9	10	10	11	11	12	12	13	14	14	14	15	15	16	17	19	20	23	26	30	38	53
1,400,000	5	6	6	6	6	6	6	6	6	7	7	7	7	7	8	8	8	8	9	9	9	10	10	11	11	12	12	13	14	14	14	15	16	16	17	19	20	23	26	30	38	53
1,380,000	5	6	6	6	6	6	6	6	6	7	7	7	7	7	8	8	8	8	9	9	9	10	10	11	11	12	12	13	14	14	14	15	16	16	17	19	20	23	26	30	38	53
1,360,000	5	6	6	6	6	6	6	6	7	7	7	7	7	7	8	8	8	9	9	9	10	10	10	11	11	12	12	13	14	14	14	15	16	16	17	19	20	23	26	30	38	53
1,340,000	5	6	6	6	6	6	6	6	7	7	7	7	7	8	8	8	8	9	9	9	10	10	10	11	11	12	12	13	14	14	14	15	16	16	18	19	20	23	26	30	38	53
1,320,000	6	6	6	6	6	6	6	6	7	7	7	7	7	8	8	8	8	9	9	9	10	10	10	11	11	12	12	13	14	14	14	15	16	17	18	19	20	23	26	30	38	53
1,300,000	6	6	6	6	6	6	6	6	7	7	7	7	7	8	8	8	8	9	9	9	10	10	10	11	11	12	12	13	14	14	14	15	16	17	18	19	20	23	26	30	38	53
1,280,000	6	6	6	6	6	6	6	6	7	7	7	7	7	8	8	8	8	9	9	9	10	10	10	11	11	12	12	13	14	14	14	15	16	17	18	19	21	23	26	30	38	53
1,260,000	6	6	6	6	6	6	7	7	7	7	7	7	8	8	8	8	8	9	9	9	10	10	11	11	11	12	12	13	14	14	14	15	16	17	18	19	21	23	26	30	38	53
1,240,000	6	6	6	6	6	6	7	7	7	7	7	7	8	8	8	8	8	9	9	9	10	10	11	11	11	12	12	13	14	14	14	15	16	17	18	19	21	23	26	30	38	53
1,220,000	6	6	6	6	6	6	7	7	7	7	7	8	8	8	8	8	9	9	9	9	10	10	11	11	12	12	12	13	14	14	15	15	16	17	18	19	21	23	26	30	38	53
1,200,000	6	6	6	6	6	6	7	7	7	7	7	8	8	8	8	8	9	9	9	10	10	10	11	11	12	12	13	13	14	14	15	15	16	17	18	19	21	23	26	30	38	53
1,180,000	6	6	6	6	6	6	7	7	7	7	7	8	8	8	8	8	9	9	9	10	10	10	11	11	12	12	13	13	14	14	15	15	16	17	18	19	21	23	26	31	38	53
1,160,000	6	6	6	6	6	6	7	7	7	7	7	8	8	8	8	8	9	9	9	10	10	10	11	11	12	12	13	13	14	14	15	15	16	17	18	19	21	23	26	31	38	53
1,140,000	6	6	6	6	6	6	7	7	7	7	7	8	8	8	8	8	9	9	9	10	10	10	11	11	12	12	13	13	14	14	15	15	16	17	18	19	21	23	26	31	38	54
1,120,000	6	6	6	6	6	7	7	7	7	7	7	8	8	8	8	9	9	9	9	10	10	10	11	11	12	12	13	13	14	14	15	15	16	17	18	19	21	23	26	31	38	54
1,100,000	6	6	6	6	6	7	7	7	7	7	8	8	8	8	8	9	9	9	9	10	10	11	11	11	12	12	13	13	14	14	15	15	16	17	18	19	21	23	26	31	38	54
1,080,000	6	6	6	6	6	7	7	7	7	7	7	8	8	8	8	9	9	9	10	10	10	11	11	11	12	12	13	13	14	14	15	16	16	17	18	19	21	23	26	31	38	54
1,060,000	6	6	6	6	7	7	7	7	7	7	8	8	8	8	8	9	9	9	10	10	10	11	11	12	12	12	13	13	14	14	15	16	16	17	18	19	21	23	26	31	38	54
1,040,000	6	6	6	6	7	7	7	7	7	7	8	8	8	8	8	9	9	9	10	10	10	11	11	12	12	13	13	14	14	15	15	16	16	17	18	19	21	23	26	31	38	54
1,020,000	6	6	6	7	7	7	7	7	7	7	8	8	8	8	8	9	9	9	9	10	10	10	11	11	12	12	13	13	14	14	15	15	16	16	17	18	20	21	23	26	31	38	54
1,000,000	6	6	6	7	7	7	7	7	7	8	8	8	8	8	9	9	9	9	10	10	11	11	11	12	12	13	13	14	14	15	15	16	17	17	18	20	21	23	26	31	38	54
980,000	6	6	7	7	7	7	7	7	8	8	8	8	8	8	9	9	9	10	10	10	11	11	11	12	12	13	13	14	14	15	15	16	17	17	18	20	21	23	26	31	38	54
960,000	6	6	7	7	7	7	7	7	7	8	8	8	8	8	9	9	9	9	10	10	10	11	11	12	12	12	13	13	14	14	15	15	16	17	17	18	20	21	23	27	31	39	54
940,000	6	7	7	7	7	7	7	7	8	8	8	8	8	9	9	9	9	10	10	10	11	11	12	12	13	13	13	14	14	15	15	16	17	18	19	20	21	24	27	31	39	54
920,000	7	7	7	7	7	7	7	7	8	8	8	8	8	9	9	9	9	10	10	10	11	11	11	12	12	13	13	14	14	15	15	16	16	17	18	19	20	22	24	27	31	39	54
900,000	7	7	7	7	7	7	7	8	8	8	8	8	8	9	9	9	9	10	10	10	11	11	11	12	12	13	13	14	14	15	15	16	16	17	18	19	20	22	24	27	31	39	54
880,000	7	7	7	7	7	7	8	8	8	8	8	8	9	9	9	9	10	10	10	10	11	11	11	12	12	13	13	14	14	15	15	16	16	17	18	19	20	22	24	27	31	39	54
860,000	7	7	7	7	7	7	8	8	8	8	8	9	9	9	9	10	10	10	10	11	11	12	12	12	13	13	14	14	15	15	16	16	17	18	19	20	22	24	27	31	39	54
840,000	7	7	7	7	8	8	8	8	8	8	8	9	9	9	9	10	10	10	11	11	11	12	12	13	13	14	14	15	15	16	16	17	17	18	19	20	22	24	27	31	39	54
820,000	7	7	7	7	8	8	8	8	8	8	9	9	9	9	9	10	10	10	11	11	11	12	12	13	13	14	14	15	15	16	16	17	17	18	19	20	22	24	27	32	39	54
800,000	7	7	7	8	8	8	8	8	8	8	9	9	9	9	9	10	10	10	10	11	11	12	12	12	13	13	14	14	15	15	16	16	17	18	18	19	21	22	24	27	32	39	55
780,000	7	7	7	8	8	8	8	8	8	9	9	9	9	9	10	10	10	11	11	11	12	12	13	13	13	14	14	15	15	16	16	17	18	18	19	21	22	24	27	32	39	55
760,000	7	8	8	8	8	8	8	8	9	9	9	9	9	10	10	10	10	11	11	11	12	12	13	13	14	14	15	15	16	16	17	17	18	19	20	21	22	24	27	32	39	55
740,000	8	8	8	8	8	8	8	9	9	9	9	9	10	10	10	10	11	11	11	12	12	12	13	13	14	14	15	15	16	16	17	17	18	19	20	21	22	25	28	32	39	55
720,000	8	8	8	8	8	8	8	9	9	9	9	9	10	10	10	10	11	11	11	12	12	13	13	14	14	14	15	15	16	16	17	17	18	19	20	21	23	25	28	32	40	55
700,000	8	8	8	8	8	9	9	9	9	9	9	10	10	10	10	11	11	11	12	12	12	13	13	14	14	15	15	16	16	17	17	18	18	19	20	21	23	25	28	32	40	55
680,000	8	8	8	8	9	9	9	9	9	9	10	10	10	10	11	11	11	11	12	12	13	13	13	14	14	15	15	16	16	17	17	18	19	19	20	21	23	25	28	32	40	55
660,000	8	8	8	9	9	9	9	9	9	10	10	10	10	10	11	11	11	12	12	12	13	13	14	14	15	15	16	16	17	17	18	18	19	20	20	22	23	25	28	33	40	55
640,000	8	9	9	9	9	9	9	10	10	10	10	10	11	11	11	11	12	12	12	13	13	13	14	14	15	15	16	16	17	17	18	18	19	20	21	22	23	25	28	33	40	55
620,000	9	9	9	9	9	9	9	10	10	10	10	10	11	11	11	11	12	12	12	13	13	14	14	15	15	16	16	17	17	18	18	19	19	20	21	22	24	26	28	33	40	55
600,000	9	9	9	9	10	10	10	10	10	10	11	11	11	11	12	12	12	13	13	13	14	14	15	15	16	16	17	17	18	18	19	19	20	21	22	24	26	29	33	40	55	
580,000	9	9	9	9	10	10	10	10	10	10	11	11	11	11	12	12	12	13	13	13	14	14	15	15	16	16	17	17	18	18	19	19	20	20	21	22	24	26	29	33	40	55
560,000	9	9	9	10	10	10	10	10	10	11	11	11	11	12	12	12	12	13	13	13	14	14	15	15	16	16	17	17	18	18	19	19	20	21	21	23	24	26	29	33	41	55
540,000	9	9	9	10	10	10	10	10	10	11	11	11	11	11	12	12	12	13	13	13	14	14	15	15	16	16	17	17	18	18	19	19	20	21	21	23	24	26	29	33	41	55
520,000	9	9	9	10	10	10	10	10	10	11	11	11	11	11	12	12	12	13	13	13	14	14	15	15	16	16	17	17	18	18	19	19	20	21	21	23	24	26	29	33	41	55
500,000	9	9	9	9	10	10	10	10	10	10	11	11	11	11	12	12	12	13	13	13	14	14	15	15	16	16	17	17	18	18	19	19	20	21	21	23	24	26	29	33	40	55
480,000	9	9	9	9	9	10	10	10	10	10	11	11	11	11	12	12	12	13	13	13	14	14	15	15	16	16	17	17	18	18	19	19	20	20	21	23	24	26	29	33	40	55
460,000	9	9	9	9	9	10	10	10	10	10	10	11	11	11	11	12	12	12	13	13	14	14	15	15	16	16	17	17	18	18	19	19	20	20	21	22	24	26	29	33	40	55
440,000	9	9	9	9	9	9	10	10	10	10	10	11	11	11	12	12	12	13	13	13	14	14	15	15	16	16	17	17	18	18	19	19	20	20	21	22	24	26	29	33	40	55
420,000	9	9	9	9	9	9	9	9	10	10	10	10	10	11	11	11	11	12	12	12	13	13	13	14	14	14	15	16	16	17	17	18	18	19	19	20	20	21	22	24	26	29	33	40	55
400,000	8	9	9	9	9	9	9	9	9	10	10	10	10	10	11	11	11	12	12	12	13	13	13	14	14	15	15	16	16	17	17	18	18	19	20	20	21	22	24	26	28	33	40	55
380,000	8	8	8	8	9	9	9	9	9	9	10	10	10	10	11	11	11	12	12	12	13	13	14	14	15	15	16	16	17	17	18	18	19	19	20	21	22	24	26	28	33	40	55
360,000	8	8	8	8	8	9	9	9	9	9	10	10	10	10	11	11	11	12	12	13	13	14	14	15	15	16	16	17	17	18	18	19	19	20	21	22	23	25	28	32	40	55
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280,000	6	6	7	7	7	7	7	8	8	8	8	8	9	9	9	10	10	11	11	11	12	13	13	14	14	15	15	16	17	17	17	18	19	19	20	21	23	24	27	31	39	54
260,000	6	6	6	7	7	7	7	7	8	8	8	8	9	9	9	10	10	10	11	11	12	12	13	14	14	15	16	16	17	17	18	18	19	19	20	21	23	25	27	32	39	54
240,000	6	6	6	6	7	7	7	7	7	8	8	8	9	9	9	10	10	10	11	11	12	13	13	14	15	15	16	16	17	17	18	18	19	20	20	21	23	25	27	32	39	54
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180,000	5	6	6	6	6	6	7	7	7	7	8	8	8	9	9	10	10	11	11	12	12	13	14	14	15	16	16	17	17	18	18	19	19	20	21	21	22	23	25	28	32	39	54
160,000	5	5	6	6	6	6	6	7	7	7	7	8	8	8	9	9	10	10	11	11	12	13	13	14	15	15	16	17	18	18	19	19	20	20	21	22	22	24	25	28	32	39	54
140,000	5	5	6	6	6	6	6	7	7	7	8	8	8	9	9	9	10	10	11	12	12	13	14	14	15	16	16	17	18	18	19	19	20	21	21	22	22	23	24	26	28	32	39	54
120,000	5	5	6	6	6	6	6	7	7	7	8	8	8	9	9	10	10	11	11	12	13	13	14	15	16	17	17	18	19	20	20	21	21	22	23	23	24	25	27	29	33	40	54
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10,000	5	5	5	5	5	5	5	6	7	8	9	10	12	13	14	16	17	18	20	22	23	25	27	29	31	34	36	38	40	41	42	43	44	45	46	47	48	49	50	51	53	65

A.2 Optimal drawdown based on balance and age (ρ = 5, ϕ = 0, female, single, homeowner, mean = 3.5%, vol = 7%)

	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100	101	102	103	104	105	106	107	108
1,500,000	5	5	5	5	5	6	6	6	6	6	6	7	7	7	7	7	8	8	9	9	9	9	10	11	11	11	11	12	14	14	14	15	15	16	17	19	20	22	26	30	38	53
1,480,000	5	5	5	5	6	6	6	6	6	6	7	7	7	7	7	8	8	9	9	9	9	10	11	11	11	12	12	14	14	14	15	15	16	17	19	20	23	26	30	38	53	
1,460,000	5	5	5	5	6	6	6	6	6	6	7	7	7	7	7	8	8	9	9	9	9	10	11	11	11	12	12	14	14	14	15	15	16	17	19	20	23	26	30	38	53	
1,440,000	5	5	5	5	6	6	6	6	6	6	7	7	7	7	7	8	8	9	9	9	9	10	11	11	11	12	12	14	14	14	15	15	16	17	19	20	23	26	30	38	53	
1,420,000	5	5	5	5	6	6	6	6	6	6	7	7	7	7	7	8	8	9	9	9	9	10	11	11	11	12	12	14	14	14	15	15	16	17	19	20	23	26	30	38	53	
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1,380,000	5	5	5	6	6	6	6	6	6	6	7	7	7	7	7	8	8	8	9	9	9	9	10	11	11	11	12	12	14	14	14	15	15	16	17	19	20	23	26	30	38	53
1,360,000	5	5	5	6	6	6	6	6	6	6	7	7	7	7	7	8	8	8	9	9	9	9	10	11	11	11	12	12	14	14	14	15	15	16	17	19	20	23	26	30	38	53
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1,320,000	5	5	5	6	6	6	6	6	6	6	7	7	7	7	7	8	8	8	9	9	9	10	10	11	11	11	12	12	14	14	14	15	16	16	18	19	21	23	26	30	38	53
1,300,000	5	5	6	6	6	6	6	6	6	6	7	7	7	7	7	8	8	8	9	9	9	10	10	11	11	11	12	12	14	14	14	15	16	17	18	19	21	23	26	30	38	53
1,280,000	5	5	6	6	6	6	6	6	6	6	7	7	7	7	7	8	8	8	9	9	9	10	10	11	11	11	12	12	14	14	14	15	16	17	18	19	21	23	26	30	38	53
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1,240,000	5	6	6	6	6	6	6	6	6	7	7	7	7	7	8	8	8	8	9	9	9	10	10	11	11	11	12	12	14	14	14	15	16	17	18	19	21	23	26	31	38	53
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1,200,000	5	6	6	6	6	6	6	6	6	7	7	7	7	7	8	8	8	8	9	9	9	10	10	11	11	12	12	13	14	14	14	15	16	17	18	19	21	23	26	31	38	53
1,180,000	6	6	6	6	6	6	6	6	7	7	7	7	7	8	8	8	8	8	9	9	9	10	10	11	11	12	12	13	14	14	14	15	16	17	18	19	21	23	26	31	38	54
1,160,000	6	6	6	6	6	6	6	6	7	7	7	7	7	8	8	8	8	9	9	9	10	10	10	11	11	12	12	13	14	14	14	15	16	17	18	19	21	23	26	31	38	54
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1,120,000	6	6	6	6	6	6	6	7	7	7	7	7	8	8	8	8	8	9	9	9	10	10	10	11	11	12	12	13	14	14	15	15	16	17	18	19	21	23	26	31	38	54
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1,080,000	6	6	6	6	6	6	7	7	7	7	7	8	8	8	8	8	8	9	9	9	10	10	11	11	11	12	12	13	14	14	15	15	16	17	18	19	21	23	26	31	38	54
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840,000	7	7	7	7	7	7	7	8	8	8	8	8	8	9	9	9	9	10	10	10	11	11	12	12	13	13	14	14	15	15	16	16	17	18	19	20	22	24	27	31	39	54
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620,000	8	8	9	9	9	9	9	9	9	10	10	10	10	10	11	11	11	12	12	12	13	13	14	14	14	15	16	16	17	17	18	18	19	20	21	22	24	26	29	33	40	55
600,000	9	9	9	9	9	9	9	10	10	10	10	10	11	11	11	11	12	12	12	13	13	13	14	14	15	15	16	16	17	17	18	19	19	20	21	22	24	26	29	33	40	55
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560,000	9	9	9	9	9	10	10	10	10	10	10	11	11	11	11	12	12	12	13	13	13	14	14	15	15	16	16	17	17	18	18	19	20	21	21	23	24	26	29	33	41	56
540,000	9	9	9	9	9	10	10	10	10	10	10	11	11	11	11	12	12	12	13	13	13	14	14	15	15	16	16	17	17	18	18	19	20	21	21	23	24	26	29	33	41	55
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320,000	7	7	7	7	7	8	8	8	8	8	9	9	9	9	10	10	10	11	11	11	12	12	13	13	14	15	15	16	16	17	18	18	19	20	21	22	23	25	28	32	39	54
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260,000	6	6	6	6	6	7	7	7	7	7	8	8	8	8	9	9	9	10	10	10	11	11	12	12	13	13	14	15	15	16	17	17	18	19	19	20	21	23	25	27	32	39	54
240,000	5	6	6	6	6	6	6	7	7	7	7	8	8	8	9	9	9	10	10	11	11	12	12	13	14	14	15	16	16	17	18	18	19	20	20	21	23	25	27	32	39	54
220,000	5	5	6	6	6	6	6	7	7	7	7	8	8	8	9	9	9	10	10	11	11	12	12	13	14	14	15	16	17	17	18	18	19	20	21	22	23	25	28	32	39	54
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180,000	5	5	5	5	6	6	6	6	7	7	7	7	8	8	9	9	9	10	10	11	11	12	13	13	14	15	16	16	17	18	18	19	20	20	21	22	23	25	28	32	39	54
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140,000	5	5	5	5	6	6	6	6	7	7	7	7	8	8	9	9	10	10	11	11	12	13	13	14	15	16	16	17	18	19	20	20	21	22	22	23	24	26	28	32	39	54
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A.3 Optimal drawdown based on balance and age ($\rho = 2$, $\phi = 0$, male, single, homeowner, mean = 3.5%, vol = 7%)

	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100	101	102	103	104	105	106	107	108
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A.4 Optimal drawdown based on balance and age ($\rho = 8$, $\phi = 0$, male, single, homeowner, mean = 3.5%, vol = 7%)

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1,460,000	5	5	5	5	5	5	5	6	6	6	6	6	6	7	7	7	7	7	9	9	9	9	9	11	11	11	11	11	14	14	14	14	14	14	15	17	18	21	24	28	36	52
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520,000	8	9	9	9	9	9	9	9	9	10	10	10	10	10	10	11	11	11	11	12	12	12	13	13	13	14	14	15	15	15	16	16	17	18	19	20	22	24	27	31	39	54
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340,000	7	7	7	7	8	8	8	8	8	8	8	9	9	9	9	9	10	10	10	10	11	11	11	12	12	12	13	13	13	14	14	15	15	16	16	17	18	19	21	23	26	30	38	53
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280,000	6	6	6	6	6	7	7	7	7	7	7	8	8	8	8	8	9	9	9	10	10	10	11	11	11	11	12	12	13	13	14	14	14	15	16	16	17	18	20	22	25	29	37	53
260,000	6	6	6	6	6	6	7	7	7	7	7	8	8	8	8	8	9	9	9	9	10	10	10	11	11	11	12	12	13	13	14	14	14	15	16	16	17	18	20	22	25	29	37	53
240,000	6	6	6	6	6	6	6	7	7	7	7	7	8	8	8	8	9	9	9	10	10	10	11	11	12	12	12	13	13	14	14	15	15	16	17	17	19	20	22	25	29	37	53
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200,000	5	6	6	6	6	6	6	7	7	7	7	7	8	8	8	8	9	9	9	10	10	10	11	11	12	12	13	13	14	14	15	15	16	16	17	18	19	20	22	25	30	37	53
180,000	5	5	6	6	6	6	6	6	7	7	7	7	8	8	8	8	9	9	9	10	10	10	11	11	12	12	13	13	14	14	15	15	16	16	17	18	19	20	22	25	30	37	53
160,000	5	5	5	6	6	6	6	6	7	7	7	7	8	8	8	8	9	9	9	10	10	10	11	11	12	12	13	13	14	14	15	15	16	16	17	17	18	19	21	22	25	30	37	53
140,000	5	5	6	6	6	6	6	6	7	7	7	8	8	8	8	8	9	9	10	10	11	11	12	12	13	13	14	14	15	15	16	16	17	17	18	19	20	21	23	26	30	37	53
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10,000	5	5	5	5	5	5	6	6	7	8	9	10	11	12	13	14	15	16	17	18	20	21	23	25	26	28	30	32	33	34	35	36	37	37	38	39	40	40	41	42	47	60

A.5 Optimal drawdown based on balance and age ($\rho = 5$, $\phi = 0$, male, single, homeowner, mean = 2.5%, vol = 7%)

	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100	101	102	103	104	105	106	107	108
1,500,000	5	5	5	5	5	6	6	6	6	6	6	7	7	7	7	7	8	8	9	9	9	9	10	11	11	11	12	12	14	14	14	14	15	16	17	18	20	22	25	30	38	53
1,480,000	5	5	5	5	5	6	6	6	6	6	7	7	7	7	7	8	8	9	9	9	9	10	11	11	11	12	12	14	14	14	14	14	15	16	17	18	20	22	25	30	38	53
1,460,000	5	5	5	5	6	6	6	6	6	6	7	7	7	7	8	8	8	9	9	9	9	10	11	11	11	12	12	14	14	14	14	14	15	16	17	18	20	22	25	30	38	53
1,440,000	5	5	5	5	6	6	6	6	6	6	7	7	7	7	8	8	8	9	9	9	9	10	11	11	11	12	12	14	14	14	14	14	15	16	17	18	20	22	25	30	38	53
1,420,000	5	5	5	5	6	6	6	6	6	6	7	7	7	7	8	8	8	9	9	9	9	10	11	11	11	12	12	14	14	14	14	14	15	16	17	18	20	22	25	30	38	53
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1,380,000	5	5	5	6	6	6	6	6	6	6	7	7	7	7	7	8	8	8	9	9	9	10	10	11	11	11	12	12	14	14	14	14	15	16	17	18	20	22	25	30	38	53
1,360,000	5	5	5	6	6	6	6	6	6	6	7	7	7	7	7	8	8	8	9	9	9	10	10	11	11	11	12	12	14	14	14	14	15	16	17	18	20	22	25	30	38	53
1,340,000	5	5	5	6	6	6	6	6	6	6	7	7	7	7	7	8	8	8	9	9	9	10	10	11	11	11	12	12	14	14	14	15	15	16	17	18	20	22	25	30	38	53
1,320,000	5	5	5	6	6	6	6	6	6	6	7	7	7	7	8	8	8	8	9	9	9	10	10	11	11	11	12	12	14	14	14	15	15	16	17	18	20	22	25	30	38	53
1,300,000	5	5	6	6	6	6	6	6	6	7	7	7	7	7	8	8	8	8	9	9	9	10	10	11	11	11	12	12	14	14	14	15	15	16	17	19	20	22	25	30	38	53
1,280,000	5	5	6	6	6	6	6	6	6	7	7	7	7	7	8	8	8	8	9	9	9	10	10	11	11	12	12	12	14	14	14	15	15	16	17	19	20	22	25	30	38	53
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1,240,000	5	6	6	6	6	6	6	6	6	7	7	7	7	7	8	8	8	8	9	9	9	10	10	11	11	12	12	13	14	14	14	15	15	16	17	19	20	22	25	30	38	53
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160,000	5	5	6	6	6	6	6	7	7	7	8	8	8	9	9	10	10	11	11	12	12	13	14	15	15	16	17	18	18	19	19	20	20	21	21	22	23	25	28	32	39	54
140,000	5	5	6	6	6	6	7	7	7	7	8	8	8	9	9	9	10	10	11	12	12	13	14	14	15	16	16	17	17	18	19	19	20	20	21	21	22	23	24	26	28	32	39	54
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A.6 Optimal drawdown based on balance and age ($\rho = 5$, $\phi = 0$, male, single, homeowner, mean = 3.5%, vol = 10%)

	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100	101	102	103	104	105	106	107	108
1,500,000	5	5	5	6	6	6	6	6	6	6	7	7	7	7	7	8	8	8	9	9	9	10	10	11	11	11	12	12	14	14	14	14	15	16	17	18	20	22	25	30	38	53
1,480,000	5	5	5	6	6	6	6	6	6	6	7	7	7	7	7	8	8	8	9	9	9	10	10	11	11	11	12	12	14	14	14	14	15	16	17	18	20	22	25	30	38	53
1,460,000	5	5	5	6	6	6	6	6	6	6	7	7	7	7	7	8	8	8	9	9	9	10	10	11	11	11	12	12	14	14	14	14	15	16	17	18	20	22	25	30	38	53
1,440,000	5	5	6	6	6	6	6	6	6	6	7	7	7	7	7	8	8	8	9	9	9	10	10	11	11	11	12	12	14	14	14	14	15	16	17	18	20	22	25	30	38	53
1,420,000	5	5	6	6	6	6	6	6	6	7	7	7	7	7	8	8	8	8	9	9	9	10	10	11	11	11	12	12	14	14	14	15	15	16	17	18	20	22	25	30	38	53
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1,380,000	5	5	6	6	6	6	6	6	6	7	7	7	7	7	8	8	8	8	9	9	9	10	10	11	11	11	12	12	14	14	14	15	15	16	17	18	20	22	25	30	38	53
1,360,000	5	6	6	6	6	6	6	6	6	7	7	7	7	7	8	8	8	8	9	9	9	10	10	11	11	12	12	12	14	14	14	15	15	16	17	19	20	22	25	30	38	53
1,340,000	5	6	6	6	6	6	6	6	6	7	7	7	7	7	8	8	8	8	9	9	9	10	10	11	11	12	12	13	14	14	14	15	15	16	17	19	20	22	25	30	38	53
1,320,000	5	6	6	6	6	6	6	6	7	7	7	7	7	7	8	8	8	9	9	9	10	10	10	11	11	12	12	13	14	14	14	15	15	16	17	19	20	22	25	30	38	53
1,300,000	6	6	6	6	6	6	6	6	7	7	7	7	7	8	8	8	8	9	9	9	10	10	10	11	11	12	12	13	14	14	14	15	15	16	17	19	20	22	25	30	38	53
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1,260,000	6	6	6	6	6	6	6	6	7	7	7	7	7	8	8	8	8	9	9	9	10	10	10	11	11	12	12	13	14	14	14	15	16	16	17	19	20	22	26	30	38	53
1,240,000	6	6	6	6	6	6	6	7	7	7	7	7	7	8	8	8	8	9	9	9	10	10	10	11	11	12	12	13	14	14	14	15	16	16	17	19	20	23	26	30	38	53
1,220,000	6	6	6	6	6	6	6	7	7	7	7	7	7	8	8	8	8	9	9	9	10	10	11	11	11	12	12	13	14	14	14	15	16	16	18	19	20	23	26	30	38	53
1,200,000	6	6	6	6	6	6	6	7	7	7	7	7	8	8	8	8	9	9	9	9	10	10	11	11	11	12	12	13	14	14	14	15	16	17	18	19	20	23	26	30	38	53
1,180,000	6	6	6	6	6	6	7	7	7	7	7	8	8	8	8	8	9	9	9	9	10	10	11	11	12	12	12	13	14	14	14	15	16	17	18	19	20	23	26	30	38	53
1,160,000	6	6	6	6	6	6	7	7	7	7	7	8	8	8	8	8	9	9	9	10	10	10	11	11	12	12	13	13	14	14	15	15	16	17	18	19	21	23	26	30	38	53
1,140,000	6	6	6	6	6	6	7	7	7	7	7	8	8	8	8	8	9	9	9	10	10	10	11	11	12	12	13	13	14	14	15	15	16	17	18	19	21	23	26	30	38	53
1,120,000	6	6	6	6	6	6	7	7	7	7	7	8	8	8	8	8	9	9	9	10	10	10	11	11	12	12	13	13	14	14	15	15	16	17	18	19	21	23	26	30	38	53
1,100,000	6	6	6	6	6	7	7	7	7	7	7	8	8	8	8	9	9	9	9	10	10	10	11	11	12	12	13	13	14	14	15	15	16	17	18	19	21	23	26	30	38	54
1,080,000	6	6	6	6	6	7	7	7	7	7	7	8	8	8	8	9	9	9	9	10	10	11	11	11	12	12	13	13	14	14	15	15	16	17	18	19	21	23	26	30	38	54
1,060,000	6	6	6	6	7	7	7	7	7	7	7	8	8	8	8	9	9	9	10	10	10	11	11	11	12	12	13	13	14	14	15	15	16	17	18	19	21	23	26	30	38	54
1,040,000	6	6	6	6	7	7	7	7	7	7	8	8	8	8	8	9	9	9	10	10	10	11	11	12	12	12	13	13	14	14	15	16	16	17	18	19	21	23	26	31	38	54
1,020,000	6	6	6	7	7	7	7	7	7	7	8	8	8	8	9	9	9	9	10	10	10	11	11	12	12	13	13	14	14	14	15	16	16	17	18	19	21	23	26	31	38	54
1,000,000	6	6	6	7	7	7	7	7	7	8	8	8	8	8	9	9	9	9	10	10	10	11	11	12	12	13	13	14	14	15	15	16	16	17	18	19	21	23	26	31	38	54
980,000	6	6	7	7	7	7	7	7	8	8	8	8	8	8	9	9	9	10	10	10	11	11	11	12	12	13	13	14	14	15	15	16	16	17	18	19	21	23	26	31	38	54
960,000	6	6	7	7	7	7	7	7	7	8	8	8	8	8	9	9	9	10	10	10	11	11	11	12	12	13	13	14	14	15	15	16	17	17	18	20	21	23	26	31	38	54
940,000	6	7	7	7	7	7	7	7	8	8	8	8	8	9	9	9	9	10	10	10	11	11	12	12	12	13	13	14	14	15	15	16	17	17	18	20	21	23	26	31	38	54
920,000	7	7	7	7	7	7	7	7	8	8	8	8	8	9	9	9	9	10	10	10	11	11	12	12	13	13	14	14	14	15	15	16	17	18	18	20	21	23	26	31	39	54
900,000	7	7	7	7	7	7	7	8	8	8	8	8	9	9	9	9	10	10	10	11	11	11	12	12	13	13	14	14	15	15	16	16	17	18	19	20	21	23	26	31	39	54
880,000	7	7	7	7	7	7	8	8	8	8	8	9	9	9	9	9	10	10	10	11	11	11	12	12	13	13	14	14	15	15	16	16	17	18	19	20	21	24	27	31	39	54
860,000	7	7	7	7	7	7	8	8	8	8	8	9	9	9	9	9	10	10	10	11	11	12	12	12	13	13	14	14	15	15	16	16	17	18	19	20	22	24	27	31	39	54
840,000	7	7	7	7	8	8	8	8	8	8	9	9	9	9	9	10	10	10	11	11	11	12	12	13	13	13	14	14	15	15	16	16	17	18	19	20	22	24	27	31	39	54
820,000	7	7	7	7	8	8	8	8	8	8	9	9	9	9	9	10	10	10	11	11	11	12	12	13	13	14	14	15	15	16	16	17	17	18	19	20	22	24	27	31	39	54
800,000	7	7	7	7	8	8	8	8	8	8	9	9	9	9	9	10	10	10	11	11	11	12	12	12	13	13	14	14	15	15	16	16	17	17	18	19	20	22	24	27	31	39	54
780,000	7	7	7	8	8	8	8	8	8	9	9	9	9	9	10	10	10	11	11	11	12	12	13	13	13	14	14	15	15	16	16	17	18	18	19	20	22	24	27	32	39	54
760,000	7	7	8	8	8	8	8	8	9	9	9	9	9	10	10	10	10	11	11	11	12	12	13	13	14	14	15	15	16	16	17	17	18	18	19	21	22	24	27	32	39	55
740,000	8	8	8	8	8	8	8	8	9	9	9	9	9	10	10	10	11	11	11	12	12	12	13	13	14	14	15	15	16	16	17	17	18	19	20	21	22	24	27	32	39	55
720,000	8	8	8	8	8	8	9	9	9	9	9	10	10	10	10	11	11	11	12	12	12	13	13	13	14	14	15	15	16	16	17	17	18	19	20	21	22	25	27	32	39	55
700,000	8	8	8	8	8	8	9	9	9	9	9	10	10	10	10	11	11	11	12	12	12	13	13	14	14	15	15	16	16	17	17	18	18	19	20	21	23	25	28	32	39	55
680,000	8	8	8	8	8	9	9	9	9	9	10	10	10	10	10	11	11	11	12	12	12	13	13	13	14	14	15	15	16	16	17	17	18	18	19	20	21	23	25	28	32	40	55
660,000	8	8	8	9	9	9	9	9	9	10	10	10	10	10	11	11	11	12	12	12	13	13	14	14	15	15	16	16	17	17	17	18	19	19	20	21	23	25	28	32	40	55
640,000	8	8	9	9	9	9	9	10	10	10	10	10	10	11	11	11	11	12	12	12	13	13	13	14	14	15	15	16	16	17	17	18	18	19	20	21	22	23	25	28	32	40	55
620,000	9	9	9	9	9	9	9	10	10	10	10	10	11	11	11	11	12	12	12	13	13	14	14	15	15	16	16	17	17	17	18	18	19	20	21	22	23	25	28	33	40	55
600,000	9	9	9	9	9	10	10	10	10	10	11	11	11	11	12	12	12	13	13	13	14	14	15	15	16	16	17	17	18	18	19	19	20	21	22	24	26	28	33	40	55	
580,000	9	9	9	9	10	10	10	10	10	10	11	11	11	11	12	12	12	13	13	13	14	14	15	15	16	16	17	17	17	18	18	19	20	20	21	22	24	26	29	33	40	55
560,000	9	9	9	10	10	10	10	10	10	11	11	11	11	11	12	12	12	13	13	13	14	14	15	15	16	16	17	17	18	18	19	19	20	20	21	22	24	26	29	33	40	55
540,000	9	9	9	9	10	10	10	10	10	10	11	11	11	11	12	12	12	13	13	13	14	14	15	15	16	16	17	17	18	18	19	19	20	20	21	22	24	26	29	33	40	55
520,000	9	9	9	9	10	10	10	10	10	10	11	11	11	11	12	12	12	13	13	13	14	14	15	15	16	16	17	17	18	18	19	19	20	20	21	22	24	26	29	33	40	55
500,000	9	9	9	9	9	10	10	10	10	10	11	11	11	11	12	12	12	13	13	13	14	14	15	15	16	16	17	17	18	18	19	19	20	20	21	22	24	26	29	33	40	55
480,000	9	9	9	9	9	10	10	10	10	10	11	11	11	11	12	12	12	13	13	13	14	14	15	15	16	16	17	17	18	18	19	19	20	20	21	22	24	26	29	33	40	55
460,000	9	9	9	9	9	9	10	10	10	10	10	11	11	11	11	12	12	12	13	13	14	14	15	15	16	16	17	17	18	18	19	19	20	20	21	22	24	26	28	33	40	55
440,000	9	9	9	9	9	9	9	10	10	10	10	11	11	11	11	12	12	12	13	13	14	14	15	16	16	17	17	18	18	18	19	20	20	21	22	24	26	28	33	40	55	
420,000	9	9	9	9	9	9	9	9	10	10	10	10	11	11	11	12	12	12	13	13	13	14	14	15	15	16	17	17	18	18	18	19	20	20	21	22	24	26	28	33	40	55
400,000	8	8	9	9	9	9	9	9	9	10	10	10	10	10	11	11	11	12	12	12	13	13	14	14	15	15	16	16	17	17	18	18	19	20	21	22	24	25	28	33	40	55
380,000	8	8	8	9	9	9	9	9	9	9	10	25	10	10	11	11	11	12	12	12	13	13	14	14	15	15	16	16	17	17	18	18	19	19	20	21	22	23	25	28	32	40	54
360,000	8	8	8	8	8	9	9	9	9	9	10	10	10	10	11	11	11	12	12	13	13	14	14	15	15	16	16	17	17	18	18	19	19	20	21	22	23	25	28	32	39	54
340,000	8	8	8	8	8	8	8	9	9	9	9	10	10	10	10	11	11	12	12	13	13	14	14	15	15	16	16	17	17	18	18	19	19	20	21	22	23	25	28	32	39	54
320,000	7	7	7	8	8	8	8	8	9	9	9	9	10	10	10	10	11	11	12	12	13	13	14	14	15	15	16	16	17	17	18	18	19	20	20	22	23	25	27	32	39	54
300,000	7	7	7	7	7	8	8	8	8	8	9	9	9	9	10	10	11	11	11	12	12	13	13	14	15	15	16	16	17	17	18	18	19	19	20	21	23	25	27	31	39	54
280,000	6	7	7	7	7	7	7	8	8	8	8	9	9	9	9	10	10	11	11	12	12	13	13	14	14	15	16	16	17	17	18	18	19	19	20	21	22	24	27	31	39	54
260,000	6	6	6	7	7	7	7	7	8	8	8	8	9	9	9	10	10	11	11	11	12	13	13	14	14	15	16	16	17	17	18	18	19	19	20	21	23	24	27	31	39	54
240,000	6	6	6	6	7	7	7	7	7	8	8	8	9	9	9	10	10	11	11	12	12	13	13	14	15	15	16	16	17	17	18	18	19	20	20	21	23	25	27	31	39	54
220,000	6	6	6	6	6	7	7	7	7	8	8	8	9	9	9	10	10	11	11	12	12	13	13	14	15	15	16	16	17	17	18	18	19	19	20	21	22	23	25	27	32	39	54
200,000	6	6	6	6	6	7	7	7	7	8	8	8	9	9	9	10	10	11	11	12	12	13	14	14	15	16	16	17	18	18	18	19	20	20	21	22	23	25	27	32	39	54
180,000	5	6	6	6	6	6	7	7	7	8	8	8	9	9	9	10	10	11	11	12	12	13	14	14	15	16	17	17	18	18	19	19	20	21	21	22	23	25	28	32	39	54
160,000	5	6	6	6	6	6	7	7	7	7	8	8	9	9	9	10	10	11	11	12	13	13	14	15	15	16	17	18	18	19	19	20	20	21	22	23	24	25	28	32	39	54
140,000	5	6	6	6	6	6	7	7	7	8	8	8	9	9	10	10	11	11	12	12	13	14	14	15	16	17	18	18	19	20	20	21	21	22	22	23	24	26	28	32	39	54
120,000	5	6	6	6	6	6	7	7	7	8	8	8	9	9	10	10	11	11	12	13	13	14	15	16	17	18	18	19	20	20	21	21	22	23	23	24	25	27	29	33	39	54
100,000	5	5	6	6	6	7	7	7	8	8	8	9	9	10	10	11	11	12	12	13	14	15	16	17	17	18	19	20	21	21	22	23	23	24	24	25	26	27	30	33	40	54
80,000	5	5	6	6	6	7	7	7	8	8	8	9	9	10	10	11	12	12	13	14	15	16	17	18	19	20	21	22	22	23	24	24	25	25	26	27	28	29	31	34	40	55
60,000	5	5	6	6	6	7	7	7	8	8	9	9	10	10	11	12	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	26	27	28	28	29	30	31	33	36	41	55
40,000	5	5	5	6	6	6	7	7	8	8	9	10	10	11	12	13	13	14	15	16	17	19	20	21	23	24	25	27	28	28	29	30	31	31	32	33	33	34	36	38	43	56
20,000	5	5	5	5	5	6	6	7	8	9	9	10	11	12	13	14	15	16	18	19	20	22	24	25	27	29	31	33	34	35	36	37	38	39	39	40	41	42	43	44	49	59
10,000	5	5	5	5	5	5	5	5	7	8	9	10	11	13	14	15	17	18	20	21	23	25	27	29	31	34	36	38	40	41	42	43	44	45	46	47	48	49	50	51	53	65

Appendix B Rules derived from optimal drawdown analysis

B.1 Rule of thumb derived from optimal drawdown analysis ($\rho = 5$, $\phi = 0$, male, single, homeowner, mean = 3.5%, vol = 7%)

	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100	101	102	103	104	105	106	107	108
1,500,000	6%	6%	6%	7%	7%	7%	7%	7%	7%	7%	7%	7%	7%	8%	8%	8%	8%	8%	9%	9%	9%	9%	9%	11%	11%	11%	11%	11%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%
1,480,000	6%	6%	6%	7%	7%	7%	7%	7%	7%	7%	7%	7%	7%	8%	8%	8%	8%	8%	9%	9%	9%	9%	9%	11%	11%	11%	11%	11%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%
1,460,000	6%	6%	6%	7%	7%	7%	7%	7%	7%	7%	7%	7%	7%	8%	8%	8%	8%	8%	9%	9%	9%	9%	9%	11%	11%	11%	11%	11%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%
1,440,000	6%	6%	6%	7%	7%	7%	7%	7%	7%	7%	7%	7%	7%	8%	8%	8%	8%	8%	9%	9%	9%	9%	9%	11%	11%	11%	11%	11%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%
1,420,000	6%	6%	6%	7%	7%	7%	7%	7%	7%	7%	7%	7%	7%	8%	8%	8%	8%	8%	9%	9%	9%	9%	9%	11%	11%	11%	11%	11%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%
1,400,000	6%	6%	6%	7%	7%	7%	7%	7%	7%	7%	7%	7%	7%	8%	8%	8%	8%	8%	9%	9%	9%	9%	9%	11%	11%	11%	11%	11%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%
1,380,000	6%	6%	6%	7%	7%	7%	7%	7%	7%	7%	7%	7%	7%	8%	8%	8%	8%	8%	9%	9%	9%	9%	9%	11%	11%	11%	11%	11%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%
1,360,000	6%	6%	6%	7%	7%	7%	7%	7%	7%	7%	7%	7%	7%	8%	8%	8%	8%	8%	9%	9%	9%	9%	9%	11%	11%	11%	11%	11%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%
1,340,000	6%	6%	6%	7%	7%	7%	7%	7%	7%	7%	7%	7%	7%	8%	8%	8%	8%	8%	9%	9%	9%	9%	9%	11%	11%	11%	11%	11%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%
1,320,000	6%	6%	6%	7%	7%	7%	7%	7%	7%	7%	7%	7%	7%	8%	8%	8%	8%	8%	9%	9%	9%	9%	9%	11%	11%	11%	11%	11%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%
1,300,000	6%	6%	6%	7%	7%	7%	7%	7%	7%	7%	7%	7%	7%	8%	8%	8%	8%	8%	9%	9%	9%	9%	9%	11%	11%	11%	11%	11%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%
1,280,000	6%	6%	6%	7%	7%	7%	7%	7%	7%	7%	7%	7%	7%	8%	8%	8%	8%	8%	9%	9%	9%	9%	9%	11%	11%	11%	11%	11%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%
1,260,000	6%	6%	6%	7%	7%	7%	7%	7%	7%	7%	7%	7%	7%	8%	8%	8%	8%	8%	9%	9%	9%	9%	9%	11%	11%	11%	11%	11%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%
1,240,000	6%	6%	6%	7%	7%	7%	7%	7%	7%	7%	7%	7%	7%	8%	8%	8%	8%	8%	9%	9%	9%	9%	9%	11%	11%	11%	11%	11%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%
1,220,000	6%	6%	6%	7%	7%	7%	7%	7%	7%	7%	7%	7%	7%	8%	8%	8%	8%	8%	9%	9%	9%	9%	9%	11%	11%	11%	11%	11%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%
1,200,000	6%	6%	6%	7%	7%	7%	7%	7%	7%	7%	7%	7%	7%	8%	8%	8%	8%	8%	9%	9%	9%	9%	9%	11%	11%	11%	11%	11%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%
1,180,000	6%	6%	6%	7%	7%	7%	7%	7%	7%	7%	7%	7%	7%	8%	8%	8%	8%	8%	9%	9%	9%	9%	9%	11%	11%	11%	11%	11%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%
1,160,000	6%	6%	6%	7%	7%	7%	7%	7%	7%	7%	7%	7%	7%	8%	8%	8%	8%	8%	9%	9%	9%	9%	9%	11%	11%	11%	11%	11%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%
1,140,000	6%	6%	6%	7%	7%	7%	7%	7%	7%	7%	7%	7%	7%	8%	8%	8%	8%	8%	9%	9%	9%	9%	9%	11%	11%	11%	11%	11%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%
1,120,000	6%	6%	6%	7%	7%	7%	7%	7%	7%	7%	7%	7%	7%	8%	8%	8%	8%	8%	9%	9%	9%	9%	9%	11%	11%	11%	11%	11%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%
1,100,000	6%	6%	6%	7%	7%	7%	7%	7%	7%	7%	7%	7%	7%	8%	8%	8%	8%	8%	9%	9%	9%	9%	9%	11%	11%	11%	11%	11%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%
1,080,000	6%	6%	6%	7%	7%	7%	7%	7%	7%	7%	7%	7%	7%	8%	8%	8%	8%	8%	9%	9%	9%	9%	9%	11%	11%	11%	11%	11%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%
1,060,000	6%	6%	6%	7%	7%	7%	7%	7%	7%	7%	7%	7%	7%	8%	8%	8%	8%	8%	9%	9%	9%	9%	9%	11%	11%	11%	11%	11%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%
1,040,000	6%	6%	6%	7%	7%	7%	7%	7%	7%	7%	7%	7%	7%	8%	8%	8%	8%	8%	9%	9%	9%	9%	9%	11%	11%	11%	11%	11%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%
1,020,000	6%	6%	6%	7%	7%	7%	7%	7%	7%	7%	7%	7%	7%	8%	8%	8%	8%	8%	9%	9%	9%	9%	9%	11%	11%	11%	11%	11%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%
1,000,000	6%	6%	6%	7%	7%	7%	7%	7%	7%	7%	7%	7%	7%	8%	8%	8%	8%	8%	9%	9%	9%	9%	9%	11%	11%	11%	11%	11%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%
980,000	6%	6%	6%	7%	7%	7%	7%	7%	7%	7%	7%	7%	7%	8%	8%	8%	8%	8%	9%	9%	9%	9%	9%	11%	11%	11%	11%	11%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%
960,000	6%	6%	6%	7%	7%	7%	7%	7%	7%	7%	7%	7%	7%	8%	8%	8%	8%	8%	9%	9%	9%	9%	9%	11%	11%	11%	11%	11%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%
940,000	6%	6%	6%	7%	7%	7%	7%	7%	7%	7%	7%	7%	7%	8%	8%	8%	8%	8%	9%	9%	9%	9%	9%	11%	11%	11%	11%	11%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%
920,000	6%	6%	6%	7%	7%	7%	7%	7%	7%	7%	7%	7%	7%	8%	8%	8%	8%	8%	9%	9%	9%	9%	9%	11%	11%	11%	11%	11%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%
900,000	6%	6%	6%	7%	7%	7%	7%	7%	7%	7%	7%	7%	7%	8%	8%	8%	8%	8%	9%	9%	9%	9%	9%	11%	11%	11%	11%	11%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%
880,000	6%	6%	6%	7%	7%	7%	7%	7%	7%	7%	7%	7%	7%	8%	8%	8%	8%	8%	9%	9%	9%	9%	9%	11%	11%	11%	11%	11%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%
860,000	6%	6%	6%	7%	7%	7%	7%	7%	7%	7%	7%	7%	7%	8%	8%	8%	8%	8%	9%	9%	9%	9%	9%	11%	11%	11%	11%	11%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%
840,000	6%	6%	6%	7%	7%	7%	7%	7%	7%	7%	7%	7%	7%	8%	8%	8%	8%	8%	9%	9%	9%	9%	9%	11%	11%	11%	11%	11%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%
820,000	6%	6%	6%	7%	7%	7%	7%	7%	7%	7%	7%	7%	7%	8%	8%	8%	8%	8%	9%	9%	9%	9%	9%	11%	11%	11%	11%	11%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%
800,000	6%	6%	6%	7%	7%	7%	7%	7%	7%	7%	7%	7%	7%	8%	8%	8%	8%	8%	9%	9%	9%	9%	9%	11%	11%	11%	11%	11%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%
780,000	6%	6%	6%	7%	7%	7%	7%	7%	7%	7%	7%	7%	7%	8%	8%	8%	8%	8%	9%	9%	9%	9%	9%	11%	11%	11%	11%	11%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%
760,000	6%	6%	6%	7%	7%	7%	7%	7%	7%	7%	7%	7%	7%	8%	8%	8%	8%	8%	9%	9%	9%	9%	9%	11%	11%	11%	11%	11%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%
740,000	6%	6%	6%	7%	7%	7%	7%	7%	7%	7%	7%	7%	7%	8%	8%	8%	8%	8%	9%	9%	9%	9%	9%	11%	11%	11%	11%	11%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%
720,000	6%	6%	6%	7%	7%	7%	7%	7%	7%	7%	7%	7%	7%	8%	8%	8%	8%	8%	9%	9%	9%	9%	9%	11%	11%	11%	11%	11%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%
700,000	6%	6%	6%	7%	7%	7%	7%	7%	7%	7%	7%	7%	7%	8%	8%	8%	8%	8%	9%	9%	9%	9%	9%	11%	11%	11%	11%	11%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%
680,000	6%	6%	6%	7%	7%	7%	7%	7%	7%	7%	7%	7%	7%	8%	8%	8%	8%	8%	9%	9%	9%	9%	9%	11%	11%	11%	11%	11%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%
660,000	6%	6%	6%	7%	7%	7%	7%	7%	7%	7%	7%	7%	7%	8%	8%	8%	8%	8%	9%	9%	9%	9%	9%	11%	11%	11%	11%	11%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%
640,000	6%	6%	6%	7%	7%	7%	7%	7%	7%	7%	7%	7%	7%	8%	8%	8%	8%	8%	9%	9%	9%	9%	9%	11%	11%	11%	11%	11%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%
620,000	6%	6%	6%	7%	7%	7%	7%	7%	7%	7%	7%	7%	7%	8%	8%	8%	8%	8%	9%	9%	9%	9%	9%	11%	11%	11%	11%	11%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%
600,000	6%	6%	6%	7%	7%	7%	7%	7%	7%	7%	7%	7%	7%	8%	8%	8%	8%	8%	9%	9%	9%	9%	9%	11%	11%	11%	11%	11%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%
580,000	6%	6%	6%	7%	7%	7%	7%	7%	7%	7%	7%	7%	7%	8%	8%	8%	8%	8%	9%	9%	9%	9%	9%	11%	11%	11%	11%	11%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%
560,000	6%	6%	6%	7%	7%	7%	7%	7%	7%	7%	7%	7%	7%	8%	8%	8%	8%	8%	9%	9%	9%	9%	9%	11%	11%	11%	11%	11%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%
540,000	6%	6%	6%	7%	7%	7%	7%	7%	7%	7%	7%	7%	7%	8%	8%	8%	8%	8%	9%	9%	9%	9%	9%	11%	11%	11%	11%	11%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%
520,000	6%	6%	6%	7%	7%	7%	7%	7%	7%	7%	7%	7%	7%	8%	8%	8%	8%	8%	9%	9%	9%	9%	9%	11%	11%	11%	11%	11%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%
500,000	8%	8%	8%	9%	9%	9%	9%	9%	9%	9%	9%	9%	9%	10%	10%	10%	10%	10%	10%	10%	10%	10%	10%	11%	11%	11%	11%	11%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%
480,000	8%	8%	8%	9%	9%	9%	9%	9%	9%	9%	9%	9%	9%	10%	10%	10%	10%	10%	10%	10%	10%	10%	10%	11%	11%	11%	11%	11%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%
460,000	8%	8%	8%	9%	9%	9%	9%	9%	9%	9%	9%	9%	9%	10%	10%	10%	10%	10%	10%	10%	10%	10%	10%	11%	11%	11%	11%	11%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%
440,000	8%	8%	8%	9%	9%	9%	9%	9%	9%	9%	9%	9%	9%	10%	10%	10%	10%	10%	10%	10%	10%	10%	10%	11%	11%	11%	11%	11%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%
420,000	8%	8%	8%	9%	9%	9%	9%	9%	9%	9%	9%	9%	9%	10%	10%	10%	10%	10%	10%	10%	10%	10%	10%	11%	11%	11%	11%	11%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%
400,000	8%	8%	8%	9%	9%	9%	9%	9%	9%	9%	9%	9%	9%	10%	10%	10%	10%	10%	10%	10%	10%	10%	10%	11%	11%	11%	11%	11%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%
380,000	8%	8%	8%	9%	9%	9%	9%	9%	9%	9%	9%	9%	9%	10%	10%	10%	10%	10%	10%	10%	10%	10%	10%	11%	11%	11%	11%	11%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%
360,000	8%	8%	8%	9%	9%	9%	9%	9%	9%	9%	9%	9%	9%	10%	10%	10%	10%	10%	10%	10%	10%	10%	10%	11%	11%	11%	11%	11%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%
340,000	8%	8%	8%	9%	9%	9%	9%	9%	9%	9%	9%	9%	9%	10%	10%	10%	10%	10%	10%	10%	10%	10%	10%	11%	11%	11%	11%	11%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%
320,000	8%	8%	8%	9%	9%	9%	9%	9%	9%	9%	9%	9%	9%	10%	10%	10%	10%	10%	10%	10%	10%	10%	10%	11%	11%	11%	11%	11%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%
300,000	8%	8%	8%	9%	9%	9%	9%	9%	9%	9%	9%	9%	9%	10%	10%	10%	10%	10%	10%	10%	10%	10%	10%	11%	11%	11%	11%	11%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%
280,000	8%	8%	8%	9%	9%	9%	9%	9%	9%	9%	9%	9%	9%	10%	10%	10%	10%	10%	10%	10%	10%	10%	10%	11%	11%	11%	11%	11%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%
260,000	8%	8%	8%	9%	9%	9%	9%	9%	9%	9%	9%	9%	9%	10%	10%	10%	10%	10%	10%	10%	10%	10%	10%	11%	11%	11%	11%	11%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%
240,000	6%	6%	6%	7%	7%	7%	7%	7%	7%	7%	7%	7%	7%	8%	8%	8%	8%	8%	9%	9%	9%	9%	9%	11%	11%	11%	11%	11%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%
220,000	6%	6%	6%	7%	7%	7%	7%	7%	7%	7%	7%	7%	7%	8%	8%	8%	8%	8%	9%	9%	9%	9%	9%	11%	11%	11%	11%	11%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%
200,000	6%	6%	6%	7%	7%	7%	7%	7%	7%	7%	7%	7%	7%	8%	8%	8%	8%	8%	9%	9%	9%	9%	9%	11%	11%	11%	11%	11%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%
180,000	6%	6%	6%	7%	7%	7%	7%	7%	7%	7%	7%	7%	7%	8%	8%	8%	8%	8%	9%	9%	9%	9%	9%	11%	11%	11%	11%	11%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%
160,000	6%	6%	6%	7%	7%	7%	7%	7%	7%	7%	7%	7%	7%	8%	8%	8%	8%	8%	9%	9%	9%	9%	9%	11%	11%	11%	11%	11%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%
140,000	6%	6%	6%	7%	7%	7%	7%	7%	7%	7%	7%	7%	7%	8%	8%	8%	8%	8%	9%	9%	9%	9%	9%	11%	11%	11%	11%	11%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%
120,000	6%	6%	6%	7%	7%	7%	7%	7%	7%	7%	7%	7%	7%	8%	8%	8%	8%	8%	9%	9%	9%	9%	9%	11%	11%	11%	11%	11%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%
100,000	6%	6%	6%	7%	7%	7%	7%	7%	7%	7%	7%	7%	7%	8%	8%	8%	8%	8%	9%	9%	9%	9%	9%	11%	11%	11%	11%	11%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%
80,000	6%	6%	6%	7%	7%	7%	7%	7%	7%	7%	7%	7%	7%	8%	8%	8%	8%	8%	9%	9%	9%	9%	9%	11%	11%	11%	11%	11%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%
60,000	6%	6%	6%	7%	7%	7%	7%	7%	7%	7%	7%	7%	7%	8%	8%	8%	8%	8%	9%	9%	9%	9%	9%	11%	11%	11%	11%	11%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%
40,000	6%	6%	6%	7%	7%	7%	7%	7%	7%	7%	7%	7%	7%	8%	8%	8%	8%	8%	9%	9%	9%	9%	9%	11%	11%	11%	11%	11%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%
20,000	6%	6%	6%	7%	7%	7%	7%	7%	7%	7%	7%	7%	7%	8%	8%	8%	8%	8%	9%	9%	9%	9%	9%	11%	11%	11%	11%	11%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%
10,000	6%	6%	6%	7%	7%	7%	7%	7%	7%	7%	7%	7%	7%	8%	8%	8%	8%	8%	9%	9%	9%	9%	9%	11%	11%	11%	11%	11%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%
0	6%	6%	6%	7%	7%	7%	7%	7%	7%	7%	7%	7%	7%	8%	8%	8%	8%	8%	9%	9%	9%	9%	9%	11%	11%	11%	11%	11%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%	14%

B.2 Sophisticated retiree rule derived from optimal drawdown analysis (ρ = 5, ϕ = 0, male, single, homeowner, mean = 3.5%, vol = 7%)

	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100	101	102	103	104	105	106	107	108
1,500,000	6	6	6	6	6	6	6	6	7	7	7	7	7	8	8	8	8	8	10	10	10	10	10	12	12	12	12	12	15	15	15	15	15	15	15	15	15	15	15	15	15	15
1,480,000	6	6	6	6	6	6	6	6	7	7	7	7	7	8	8	8	8	8	10	10	10	10	10	12	12	12	12	12	15	15	15	15	15	15	15	15	15	15	15	15	15	15
1,460,000	6	6	6	6	6	6	6	6	7	7	7	7	7	8	8	8	8	8	10	10	10	10	10	12	12	12	12	12	15	15	15	15	15	15	15	15	15	15	15	15	15	15
1,440,000	6	6	6	6	6	6	6	6	7	7	7	7	7	8	8	8	8	8	10	10	10	10	10	12	12	12	12	12	15	15	15	15	15	15	15	15	15	15	15	15	15	15
1,420,000	6	6	6	6	6	6	6	6	7	7	7	7	7	8	8	8	8	8	10	10	10	10	10	12	12	12	12	12	15	15	15	15	15	15	15	15	15	15	15	15	15	15
1,400,000	6	6	6	6	6	6	6	6	7	7	7	7	7	8	8	8	8	8	10	10	10	10	10	12	12	12	12	12	15	15	15	15	15	15	15	15	15	15	15	15	15	15
1,380,000	6	6	6	6	6	6	6	6	7	7	7	7	7	8	8	8	8	8	10	10	10	10	10	12	12	12	12	12	15	15	15	15	15	15	15	15	15	15	15	15	15	15
1,360,000	6	6	6	6	6	6	6	6	7	7	7	7	7	8	8	8	8	8	10	10	10	10	10	12	12	12	12	12	15	15	15	15	15	15	15	15	15	15	15	15	15	15
1,340,000	6	6	6	6	6	6	6	6	7	7	7	7	7	8	8	8	8	8	10	10	10	10	10	12	12	12	12	12	15	15	15	15	15	15	15	15	15	15	15	15	15	15
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120,000	6	6	6	6	6	6	6	6	8	8	8	8	8	10	10	10	10	10	13	13	13	13	13	15	15	15	15	15	17	17	17	17	17	17	17	17	17	17	17	17	17	17
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B.3 Financial adviser rule based on optimal drawdown analysis ($\rho = 5$, $\phi = 0$, male, single, homeowner, mean = 3.5%, vol = 7%)

	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100	101	102	103	104	105	106	107	108
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1,480,000	6	6	6	7	7	7	7	7	7	7	8	8	8	8	9	9	9	9	10	10	10	11	11	12	12	13	13	14	14	14	14	14	14	14	14	14	14	14	14	14	14	14
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1,440,000	6	6	6	7	7	7	7	7	7	7	8	8	8	8	9	9	9	9	10	10	10	11	11	12	12	13	13	14	14	14	14	14	14	14	14	14	14	14	14	14	14	14
1,420,000	6	6	6	7	7	7	7	7	7	7	8	8	8	8	9	9	9	9	10	10	10	11	11	12	12	13	13	14	14	14	14	14	14	14	14	14	14	14	14	14	14	14
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1,380,000	6	6	6	7	7	7	7	7	7	7	8	8	8	8	9	9	9	9	10	10	10	11	11	12	12	13	13	14	14	14	14	14	14	14	14	14	14	14	14	14	14	14
1,360,000	6	6	6	7	7	7	7	7	7	7	8	8	8	8	9	9	9	9	10	10	10	11	11	12	12	13	13	14	14	14	14	14	14	14	14	14	14	14	14	14	14	14
1,340,000	6	6	6	7	7	7	7	7	7	7	8	8	8	8	9	9	9	9	10	10	10	11	11	12	12	13	13	14	14	14	14	14	14	14	14	14	14	14	14	14	14	14
1,320,000	6	6	6	7	7	7	7	7	7	7	8	8	8	8	9	9	9	9	10	10	10	11	11	12	12	13	13	14	14	14	14	14	14	14	14	14	14	14	14	14	14	14
1,300,000	6	6	6	7	7	7	7	7	7	7	8	8	8	8	9	9	9	9	10	10	10	11	11	12	12	13	13	14	14	14	14	14	14	14	14	14	14	14	14	14	14	14
1,280,000	6	6	6	7	7	7	7	7	7	7	8	8	8	8	9	9	9	9	10	10	10	11	11	12	12	13	13	14	14	14	14	14	14	14	14	14	14	14	14	14	14	14
1,260,000	6	6	6	7	7	7	7	7	7	7	8	8	8	8	9	9	9	9	10	10	10	11	11	12	12	13	13	14	14	14	14	14	14	14	14	14	14	14	14	14	14	14
1,240,000	6	6	6	7	7	7	7	7	7	7	8	8	8	8	9	9	9	9	10	10	10	11	11	12	12	13	13	14	14	14	14	14	14	14	14	14	14	14	14	14	14	14
1,220,000	6	6	6	7	7	7	7	7	7	7	8	8	8	8	9	9	9	9	10	10	10	11	11	12	12	13	13	14	14	14	14	14	14	14	14	14	14	14	14	14	14	14
1,200,000	6	6	6	7	7	7	7	7	7	7	8	8	8	8	9	9	9	9	10	10	10	11	11	12	12	13	13	14	14	14	14	14	14	14	14	14	14	14	14	14	14	14
1,180,000	6	6	6	7	7	7	7	7	7	7	8	8	8	8	9	9	9	9	10	10	10	11	11	12	12	13	13	14	14	14	14	14	14	14	14	14	14	14	14	14	14	14
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