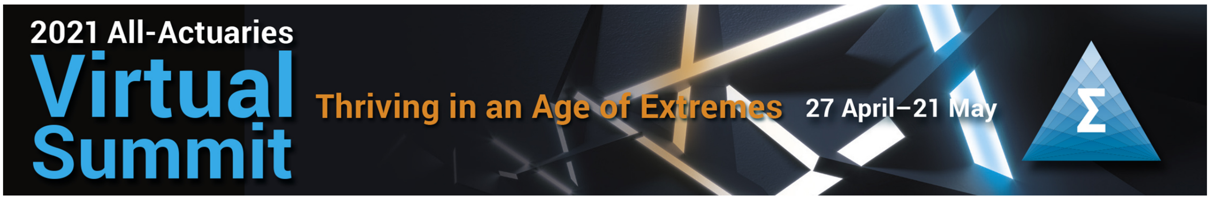

Australian Investment Performance 1959 to 2021 (and Investment Assumptions for Stochastic Models)

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Abstract

This paper analyses 62 years, or 248 quarters, of Australian investment performance from 30 June 1959 (and earlier for some sectors) to 30 June 2021. The final 248th quarter is based on 31 March 2021 market values and 31 March 2021 annual rates for financial indicators.

The paper updates previous papers presented in 2005, 2007, 2009, 2013 and 2017 to the Institute of Actuaries of Australia and to the ICA 2010, 2014 and 2018. The aim is to assess whether the methodology for determining assumptions in the previous papers is still robust enough to produce reasonable financial assumptions:

(a) now that a further four years of financial data, culminating with the coronavirus global pandemic, is being examined, and

(b) satisfying the additional challenges and insights now possible, from extending the maximum projection period from 40 years to 100 years.

The analysis covers eleven investment classes and four key financial indicators:

Growth Securities	**Interest Income**	**Financial Indicators**
Australian shares	Australian fixed interest	CPI (price inflation)
Int'l shares (hedged)	Int'l fixed interest (hedged)	AWOTE (wage inflation)
Int'l shares (unhedged)	Government semis (0-3yrs)	90-day bill rates
Property trusts	Inflation linked bonds	10-year bond rates
Direct property	Loans/corporate credit	
	Cash	

For each of these 15 "sectors" the annualised average results are tabulated and summarised for:

- risk margins (over 10-year bond rates),
- coefficients of variation,
- skewness,
- kurtosis,
- cross-correlations, and
- auto-correlations.

From these results, assumptions are developed for the mean, standard deviation, skewness, kurtosis, cross-correlations and auto-correlations for each sector. The assumptions are intended for both medium-term (3 to 10-year) and long-term (10 to 100-year) modelling. These assumptions are primarily designed for use, until about 2024, in stochastic investment and asset/liability modelling. After about two or three years they should be updated.

The paper also analyses:

- economic cycles using a sine curves technique,
- the impact of the Global Financial Crisis, comparing it, and subsequent recoveries, with previous market downturns in 1974, 1987 and 2002/03, and
- auto-correlations for both Australian share returns and 10-year bond rates.

Keywords: *investment assumptions, stochastic, skewness, kurtosis, cross-correlations, auto-correlations, cycles*

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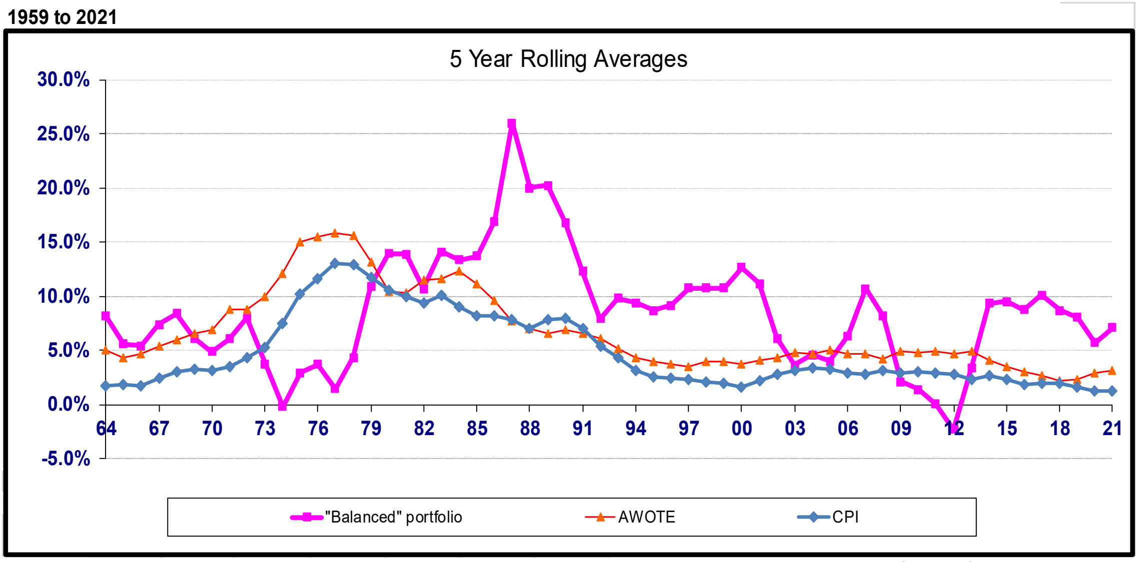
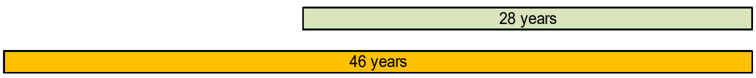


Historical 5-year (ending 30/6/64 to 30/6/21) compound average annual returns and rates Source: *Austmod*, net of tax and fees



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1 Introduction

1.1 **Demand:** Most actuarial work requires assumptions about future investment returns. Often, assumptions are also explicitly or implicitly made about future rates of inflation and/or salary increases.

1.2 **Supply:** However, often for commercial reasons, little information is publicly available for many sectors/asset classes about the historical data from which investment return assumptions, particularly long-term assumptions, can be derived.

1.3 This paper attempts to bridge, at least partly, the gap between demand and supply.

1.4 Actuaries use investment assumptions in all practice areas for:

(a) premium or contribution rate calculations,
(b) valuations of liabilities,
(c) (sometimes) valuations of assets,
(d) capital assessments,
(e) benefit and other projections, and
(f) (often) investment strategy calculations.

1.5 In February 2009 the IAAust Life Insurance and Wealth Management Practice Committee announced the establishment of an Equity Risk Premium (ERP) Research Group whose objective is to "assist actuaries in forming their view of methods to determine the equity risk premium for use in practise". The explanatory note announcing the plans for the ERP Research Group commenced:

> "***BACKGROUND***
>
> *1 The equity risk premium in which we are interested is the additional return (over government bonds of similar duration) that can be expected to be earned from investment in shares – to compensate for non-diversifiable risk. It is the only significant variable that cannot be observed or inferred from market prices.*
>
> *2 Judgements as to the size of the equity risk premium, however, form an essential element of a number of actuarial tasks:*
>
> - *Valuation of risky cash flows that are not linked to listed assets or liabilities (such as with Embedded or Appraisal Values)*
> - *All investment strategy problems where the liabilities are not guaranteed (such as the selection of benchmarks and asset liability management)*
> - *Determination of capital required when guaranteed liabilities are backed by equity type assets*
>
> *3 Obtaining the best estimate of the equity risk premium is a public good. There is limited competitive advantage in being able to predict the risk premium accurately."*

1.6 Though important, the ERP is just one assumption for each share sector (Australian Shares and/or International Shares). This paper aims to set many assumptions - for means, standard deviations, skewness, kurtosis, cross-correlations and auto-correlations - for each of the 15 sectors under consideration.

2 Methodology

2.1 **The methodology for determining assumptions seeks to determine a best estimate of the n-year first, second, third and fourth moments ending two years after the end of the data period.** The one methodology is used, with only slight variations, for calculating assumptions for risk margins, coefficients of variation, skewness, kurtosis and cross and auto correlations. The methodology involves six steps.

2.2 **Step 1:** annual rates of return and forces of return for the 15 sectors were sorted into four separate arrays:

- **(i)** years ending 30 September,
- **(ii)** years ending 31 December,
- **(iii)** years ending 31 March, and
- **(iv)** years ending 30 June.

2.3 **Step 2:** determine the periods ("n years") over which statistics are to be calculated. For the first and second moments periods of 46 years and 28 years were chosen for the reasons explained in Section 5 (in particular sections 5.18 to 5.21). The third and fourth moments were based solely on the longer 46-year period.

2.4 **Step 3:** four 46-year periods (one from each array in step 1 above) were used to derive average annual risk margins, coefficients of variation, skewness, kurtosis and 105 ranked and unranked cross-correlations. These were calculated and tabulated for each of the 46-year running periods for each quarter ending in the sixteen years between 30/9/05 and 30/6/21 inclusive. Average annual risk margins and coefficients of variation were also calculated and tabulated for 28-year periods.

2.5 **Step 4:** the four quarter-ending results were averaged to give sets of sixteen 46-year running period results. That is:

Table 2.1 Periods for 46-year calculations

Period	Average of four periods ending:
15	30/9/05, 31/12/05, 31/3/06 and 30/6/06
14	30/9/06, 31/12/06, 31/3/07 and 30/6/07
13	30/9/07, 31/12/07, 31/3/08 and 30/6/08
12	30/9/08, 31/12/08, 31/3/09 and 30/6/09
11	30/9/09, 31/12/09, 31/3/10 and 30/6/10
10	30/9/10, 31/12/10, 31/3/11 and 30/6/11
9	30/9/11, 31/12/11, 31/3/12 and 30/6/12
8	30/9/12, 31/12/12, 31/3/13 and 30/6/13
7	30/9/13, 31/12/13, 31/3/14 and 30/6/14
6	30/9/14, 31/12/14, 31/3/15 and 30/6/15
5	30/9/15, 31/12/15, 31/3/16 and 30/6/16
4	30/9/16, 31/12/16, 31/3/17 and 30/6/17
3	30/9/17, 31/12/17, 31/3/18 and 30/6/18
2	30/9/18, 31/12/18, 31/3/19 and 30/6/19
1	30/9/19, 31/12/19, 31/3/20 and 30/6/20
0	30/9/20, 31/12/20, 31/3/21 and 30/6/21

For example, as at 30 June 2021:

0 represents periods ending between 0 and 0.75 years ago, and
9 represents periods ending between 9 and 9.75 years ago.

2.6 The same periods (15 to 0) were used for the 28-year calculations. For example, in these calculations period 15 represents the average of 28-year calculations ending 30/9/05, 31/12/05, 31/3/06 and 30/6/06.

2.7 **Step 5:** a weighted quadratic EXCEL trend function was fitted to each of the 16 results from section 2.5 (and to each of the 16 results from section 2.6) and projected forward six years. For the 46-year calculations the weights used were 20% for the earliest period, increasing linearly to 100% for period 0. A quadratic basis was chosen for the fitting to avoid spurious turning-points. For the cross and auto correlations the approach was slightly varied (see Sections 13 and 15). The numbers of fittings and projections were:

Table 2.2 Number of fittings and projections

Projection	Fittings & projections	Number	Bases
Risk margins	Two per sector except bonds	28	28 & 46 yrs
Coeff. of variation	Two per sector	30	28 & 46 yrs
Skewness	One per sector	15	46 years
Kurtosis	One per sector	15	46 years
Cross-correlations	One per combination	105	46 years
Rank cross-correlns.	One per combination	105	46 years
Auto-correlations	38 for shares	38	40 years
Auto-correlations	58 for bonds	58	60 years

For the 28-year calculations, that were only used for risk margin and coefficient of variation calculations, the trend weights used were 6.25% (i.e. 100%/16) for the earliest period, increasing linearly to 100% for period 0. Thus, for these calculations, data before the year ending 30/9/78 was not used and data before the year ending 30/9/85 was given a weight of less than 50%. This was done to achieve three objectives:

(a) Importantly, by starting with a low trend weight (i.e. 20% or 6.25%) the impact on running averages, period by period, from dropping off old data was minimised – thus the trends in the running averages are impacted far more by the new data being introduced each year and far less by the old data dropping off. This is important for the Step 6 two-year projection (or extrapolation).

(b) For the 28-year calculations it minimised the use of old data, some of which is "backdated" (see Section 4).

(c) By using 46-year averages with trend weights starting at 20% (rather than the 6.25% used for the 28-year averages), the difference between both sets of results was increased – so as to give a range of possible assumptions for the important risk margin and coefficient of variation calculations.

2.8 **Step 6:** recommendations of medium to long-term assumptions were made based primarily, but not totally, on the two-year projected results. **The rationale for this is that these can be considered as a 'best estimate' of the 46-year (or 28-year) four-quarter averages expected in two years' time.** The two years might also be considered as the possible approximate lifetime of these assumptions until they are subsequently updated. **This weighted trend plus two-year approach has been designed to give greatest weight to the latest period with recognition of past trends assumed to continue for at least the next two years.** The projected results for year 2 were used as a guide (or "pointer") to set the "NEW" assumptions (refer section 6.11) for the full term of the stochastic (or deterministic if applicable) modelling.

2.9 Pragmatically, the trend plus two-year approach should also provide a reasonable response to the critic who argues that, because of such-and-such recent changes, use of (unadjusted) historical results is not appropriate for setting financial assumptions. The validity of the methodology increases if there are no discernible significant data changes over the past 62 years that are "outside the norm". Also, because Step 4 is based on running averages their trends (in Step 5) are impacted by the new data being introduced each year as well as by the old data dropping off. For the 46-year averages the old data dropping-off commences for the year ending 30 September 1960 and ends for the year ending 30 June 1975. However, for 7 of 15 sectors, the last two years of this 15.75-year period were "outside the norm" (section 3.1 explains the sector codes):

L sector, year-ending 30/9/73	lowest annual return in 62 years	0.02%
F sector, year-ending 30/9/73	lowest annual return in 62 years	-20.14%
G sector, year-ending 30/9/73	lowest annual return in 62 years	-4.46%
J sector, year-ending 30/9/73	lowest annual return in 62 years	-15.02%
H sector, year-ending 30/9/74	lowest annual return in 62 years	-40.05%
W sector, year-ending 31/3/75	highest annual rate in 62 years	29.37%
X sector, year-ending 31/3/75	highest annual rate in 62 years	17.60%

For this reason, the lowest trend weight used in the 46-year calculations was set at 20% rather than the 50% used in the 44-year calculations in Grenfell and Sneddon (2017).

2.10 The projected results for years 3 to 6 of the projection were used:

- to check that the results are not trending to ridiculous levels such as negative coefficients of variation or correlations greater than 0.99, and
- to get 'smooth' correlation assumptions at 2 years by averaging the results for periods 0 to 4.

2.11 Consistent with accepted practice in Australia for input to stochastic projection models the following were calculated and tabulated:

(a) risk margins and means based on **arithmetic** averages of rates
(b) coefficients of variation and standard deviations based on **rates**
(c) skewness, kurtosis and correlations based on **forces**.

The inputs in (a) and (b) are based on arithmetic averages and rates so that they can be readily understood and judged by fund managers, economists, investment advisors and others who often seem to prefer and be familiar with such terms (though many, including myself do not support the practice – for example, the use of rates in (b) above can potentially hide risky sectors from proper consideration). These inputs are converted to forces for stochastic model calculations and then converted back to rates at the output stage. With the *Austmod* stochastic investment simulation model the output includes both compound and arithmetic averages. The possibility of inconsistencies arising from (a), (b) or (c) is discussed in Section 11.

2.12 Once assumptions relating to the results in section 2.11 (a), (b) and (c) above have been set, it is of course possible to calculate equivalent compound average rates (and forces) for each sector.

2.13 A deliberate consequence of section 2.5 above is that all individual results quoted in Sections 6 to 8 and 10 to 15 are the average of 4 results, **not** individual results for 46 or 28 years ending on one date.

3 The 15 Sectors

3.1 The 15 "sectors" included in the analysis are:

Table 3.1 Sector descriptions

B	Bill rate	Bill rate (90-day bank) in middle of year
C	Cash	Bloomberg Ausbond Bank Bill Index, Cash sector NM/AXA No. 2 Fund pre 30/6/09
D	Bond rate	10-year bond rate in middle of year
F	Fixed interest	Bloomberg Ausbond Govt 0+ yrs Index, NM/AXA No. 2 Fund pre 30/6/09 (prev. G)
G	Govt semi	Government semis 0-3 years (SBC/UBS Warburg index SSG03)
H	Int'l shares (h)	MSCI World ex Australia, net dividends reinvested (ndr), hedged
I	Int'l shares (uh)	MSCI World ex Aust (ndr) unhedged, NM/AXA pre 30/6/09 (MSCIAI pre 30/6/88)
J	Int'l fixed	FTSE World Govt Bond index, NM/AXA Int'l Bonds pre 30/6/09, hedged
L	Loans/credit	Bloomberg Ausbond Credit 0+ yrs Index, Loans NM/AXA No. 4 Fund pre 30/6/09
N	Inflation linked	Bloomberg Ausbond Inflation-linked bonds (all maturities) Index
P	Property	Mercer Unlisted Property, one-third NM/AXA, two-thirds AMP No. 2 pre 30/6/09
Q	Prop trust	Property trust accumulation index (from 31/1/01 S&P/ASX 300, from 30/6/02 GICS)
S	Shares	S&P/ASX 300 AI, NM/AXA No. 2 Fund pre 30/6/09 (All Ordinaries AI pre 31/3/65)
W	AWOTE	AWOTE by quarter (= average 1.5 months lag), not seasonally adjusted. Full-time adults (post 9/81), Males original (pre 9/81), AWE Males (pre 1/75)
X	CPI	CPI index by quarter end

3.2 The codes B to X above are used throughout the paper to denote each of the sectors. The sectors in red font above constitute the financial indicator data.

3.3 The 15 sectors comprise 5 growth (or "equity") sectors (H, I, P, Q and S), 6 interest income sectors (C, F, G, J, L and N) and 4 financial indicators (B, D, W and X). The main database contains **annual forces at quarterly intervals**.

3.4 The financial indicator data has been collected at the same intervals and for the same periods as the investment data because they are useful in both deterministic and stochastic models for valuing, projecting and/or illustrating liabilities.

3.5 Bills and bonds are taken at mid-year because this gives a far better correlation against annual cash and loan rates. The lead between bond rates and annual investment returns for the F sector (Australian fixed interest) is considered in Section 6. The unstable lag between bond rates and CPI and AWOTE is considered in Section 14.

3.6 The November 2020 AWOTE was published in mid-February 2021 and was 1,711.60 (for the "original", "full-time adult" series). This value was used in all calculations for the year ending 31 December 2020. All "sector W" calculations involve an implicit average lag of 1.5 months (e.g. mid-May to end-June) as compared with AWOTE. Since the mid-May 2012 AWOTE, the index publishing cycle has been changed to half-yearly, so estimates have been used to obtain 2012 to 2021 values for the months of February and August.

4 "Backdating"

4.1 The available data for the various sectors commence from different dates. The following is a summary of the historical start dates for **quarterly** data (for bonds and CPI, yearly data are available from earlier dates, and for some other sectors, one or two months' data is available prior to the date shown).

Table 4.1 Historical start dates for data series

Series Start Date	Sector	Data series
30/9/41	W	AWE all males, total earnings
30/9/48	X	CPI (by quarter end)
1950	S	All Ordinaries unweighted average dividend yield
31/3/58	D	Bond rate
31/3/58	S	All Ordinaries price index
31/12/59	B	13-week treasury notes (see section 4.7)
31/3/65	S	Australian equities (EFG system) (see section 4.3)
31/3/65	F	Australian government securities (EFG system "G" sector)
30/9/69	B	90-day bank bills
30/6/71	P	AMP No. 2 Fund property sector
31/12/71	P	Property (EFG system)
31/12/71	S	Australian shares (EFG system)
1974	S	All Ordinaries weighted average dividend yield
31/12/74	W	AWOTE males, ordinary time earnings
31/3/77	Q	Listed property trust accumulation index
30/9/79	C	Cash (EFG system)
30/9/81	W	AWOTE full-time adults, ordinary time, not seasonally adjusted
30/9/82	I	International shares (EFG system, see section 4.7)
30/9/85	F	Australian fixed interest (EFG system, previously "G" sector)
30/6/86	J	International bonds (ceased 30/6/87, recommenced 30/6/92)
30/6/86	L	Loans (market valued, NM/AXA No. 4 Fund)
31/12/89	G	Semi-government bonds: SBC index (0 to 3 years)
31/3/91	N	Inflation-linked bonds: WDR index (all maturities)
31/3/00*	H	Hedged international shares: MSCI World ex Australia index
30/6/09**	S	Australian shares: S&P/ASX 300 accumulation index
30/6/09**	I	Unhedged international shares: MSCI World ex Australia index
30/6/09**	P	Mercer unlisted Australian property pre-tax asset weighted index
30/6/09**	L	Corporate credit: UBS Credit 0+ yrs index
30/6/09**	F	UBS Australian Government Bond 0+ yrs Index
30/6/09**	J	Citigroup World Government Bond Index
30/6/09**	C	UBS Australian bank bill index

* This index commenced before 31/3/00 but the earlier index values are not widely published.

** These indices commenced before 30/6/09 but the earlier index values have not been used in the *Austmod* historical database; this gives consistency with the database used in Grenfell (2009).

4.2 It is evident from the above summary that data definitions for nine sectors, i.e. W, S, B, F, P, I, L, J and C have changed over time, though apart from the 30 June 2009 changes most of these changes relate to prior 1985.

4.3 It is also evident from the above summary that the National Mutual/AXA Australia, EFG ("E" for equities, "F" for fixed interest and "G" for government) investment unitisation system started on 31 March 1965. Sales brochures in the 1970s described it in these terms:

> *"Of major significance was the introduction in 1965 of a selective investment facility known as the EFG system. Evidence of the success and wide acceptance of this concept, which was pioneered by National Mutual in Australia, may now be seen in the fact that it has since been adopted by a number of other financial institutions as a medium for superannuation investment."*

4.4 There can be little doubt that the EFG system provoked the rapid development of the now enormous managed fund industry and the extensive use of unitisation throughout the Australian superannuation industry.

4.5 It is desirable with stochastic investment models to have consistency between assumptions both within sectors (e.g. risk margins and standard deviations) and across sectors (e.g. cross-correlations). An efficient way to achieve such consistency is to have a 'complete' database for all sectors back to the one date, hence the term "backdating".

4.6 The chosen common start date for the complete database was 30 June 1959. The backdating was achieved primarily but not solely by the method of least squares, based on fitted parameters determined from data **after** the respective start dates and applying them to known data for other 'like' sectors **before** these start dates.

4.7 For each sector, the formulae used to backdate the annual forces prior to the start dates in Table 4.1, were:

$Q = 52.06\%F + 30.42\%S + 14.40\%C + 6.42\%L$

$P = 88.58\%C + 50.02\%X - 23.89\%F$

$L = 89.89\%C + 21.23\%F + 1.50\%P$

$G = 74.84\%C + 37.27\%F$

$J = 76.74\%F + 19.25\%C$

$C = 22.68\%B_{-2} + 27.44\%B_{-1} + 22.82\%B + 25.76\%B_{+1}$
where B_t = B in t quarter's time

$N = 71.38\%X + 62.99\%F - 195.05\%d$
where d = delta D force

B = in nominal terms, 13-week Treasury Note rate + 1.37%
where 1.37% = median excess of 90-day bank bill rate over 13-week Treasury Note rate from 30/9/69 to 30/9/79.

13-week Treasury Note rates are available from 30/11/59. Rates for 30/6/59 and 30/9/59 were estimated by assuming that the ratio of 13-week Treasury Note rates to 2-year Government Security rates at these dates were the same as the ratio of 13-week rates to 2-year rates at 30/11/59. (2-year Government Security rates are available from 31/1/58.)

F = 87.09%D + 14.33%B - 673.02%d

S = approximate All Ordinaries accumulation index prior 31/3/65 with June values as published by the RBA. September, December and March values based on All Ordinaries price indices plus unweighted average dividend yields less 1.75% per annum (being the average difference between unweighted and weighted dividend yields between 1974 and 1984).

I = MSCI accumulation index from 30/6/70 to 30/6/88 (the EFG system I sector data from 30/9/82 to 30/6/88 was disregarded because for some of this period the sector was partially currency-hedged). Prior to 30/6/70 the S&P500 series plus an assumed 3% per annum average dividend yield was used. All index returns were then adjusted for \$AU/\$US exchange rate movements.

H = 108.54%I + 80.97%ln(E_0/E_{-4}) + 1.81%

where E_t = \$AU/\$US exchange rate t quarters after year-end.

4.8 The process of creating historical data by regression analysis means that several series must be correlated, at least for the period over which the data is created. The correlation coefficients between the series for any 46-year (or 28-year) period that extends back into the “backdated” period must therefore be illusionary in part. It is very difficult to unravel this effect. An examination was made to try and judge whether any significant bias was introduced. For the six most affected sectors (i.e. G, J, L, N, P and Q) 13 cross-correlations were calculated over the full period since 31/3/1965 to the calculation date and over the shorter periods from when each of these sectors commenced up to the calculation date. Each of these was compared with the relevant backdating parameter used to calculate the “backdated data”. The analysis indicated that the parameters introduced very little, if any, bias overall.

4.9 Any bias introduced will also ‘wear off’ over time due to the trend-fitting, weighting and extrapolation inherent in the methodology described in Section 2. Nevertheless, users of the *Austmod* investment simulation model (refer Section 22 and Appendix C), and users of the cross-correlations assumptions tabulated in Sections 14 and 16, need to be aware that the “backdating” may have introduced some slight bias for those sectors where actual historical data started at later dates (see Table 4.1).

5 Economic Cycles

5.1 This section considers economic cycles. Though closely related, matters relating to auto-correlations are considered later, in Section 15. There are important reasons for identifying and considering economic cycles 'up-front'. The length of **past** economic cycles is a key determinant for setting the periods over which statistics are calculated for the purposes of "Step 2" in section 2.3. Also, examining the persistency and characteristics of past economic cycles:

(1) helps the researcher understand the data, and

(2) when setting assumptions, guards against the danger of confusing a long-term trend with the down-side or upside of a cycle, and

(3) may indicate the need to allow for **future** economic cycles in long-term assumptions, and

(4) is essential for setting initial cycle positions if the long-term assumptions allow for cycles or auto-correlations.

5.2 Spurred by the Global Financial Crisis, the International Actuarial Association (IAA) has been active in addressing issues relating to the management and oversight of financial services industries. They have recommended that existing prudential capital requirements need to become **more dynamic and counter-cyclical**, rather than pro-active. A precursor for this initiative to be effective is to have robust and dynamic techniques for identification of economic cycles.

5.3 Dwonczyk (1993) identified 33-year cycles in Australian CPI inflation data. "A sine wave was fitted to the data which best estimated the actual [yearly rates] time series for the period 1956 to 1992". He graphed the actuals, seven year moving averages and time series estimates over the 91-year period from 1 July 1901 to 30 June 1992.

5.4 For comparison, using the data described in Section 3 above with annual forces at quarterly intervals, the following curve-fitting results were obtained. Some curves are single others are combined. Each combined curve is **the sum of two sine curves**. The period of each sine curve is shown in Table 5.1 and only optimum results (with some minor rounding) are tabulated.

5.5 The start-point for CPI (year-ending September 1949) corresponds with when quarterly data first became available. The same start-point was used for AWOTE (but see section 5.14). The start-point for Bonds (31 March 1958) has been extended back to 30 June 1950 by interpolating between known 30 June values to estimate intervening 30 September, 31 December and 31 March values.

5.6 To give greater significance to more recent data, the data behind Table 5.1 was discounted on a compound basis for each quarter prior 30 June 2021. Thus, for example, with a data discount of 0.5% per quarter, data before December 1975 was given a weight of less than 40% and data before March 2000 was given a weight of less than 65%. By changing the data discount rate, it is possible to get an indication of whether or not results are period dependent – in the following, results are shown for both a zero discount and for a discount of 0.5% per quarter.

5.7 Table 5.1 updates the corresponding tables in Grenfell (2009, 2013 and 2017). For Bonds, results from Grenfell (2017) are included for comparison.

5.8 The Institute of Actuaries of Australia Taskforce (2005), which was established to provide a tenth anniversary review of the Resilience Reserves used in the determination of statutory solvency and capital adequacy requirements of life insurance companies in Australia, stated in Section 4 of their report:

"Actuarial intuition, reflected in almost all actuarial models, is that dividend yields, real interest rates and inflation show a tendency to revert to some longer term mean. If true, this means that the probability of a future adverse shock depends, at least in part, upon the current state of the market relative to its mean position."

In this context, it is relevant to note that single sine curves (plus random noise) are equivalent to mean reversion.

Table 5.1 Economic cycles – fitted sine curves

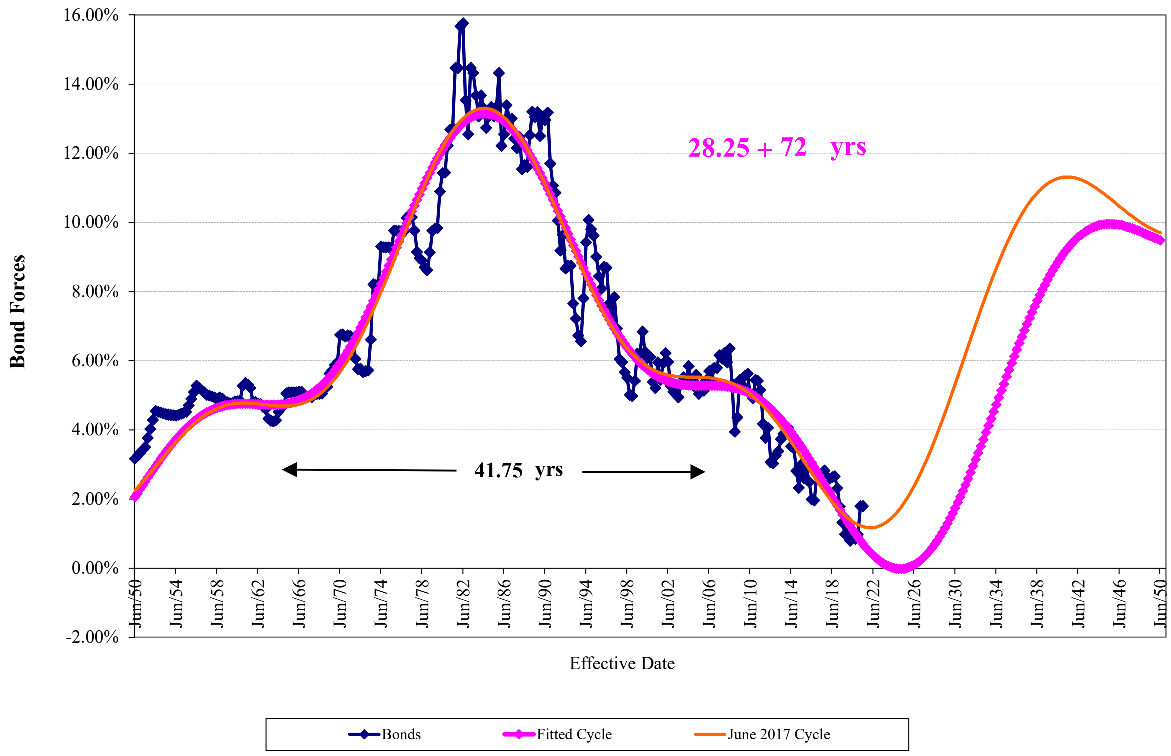
Periods of fitted sine curve cycles (years)	**Explained % of sum of squared residuals**				**Fitted curve mini-mum date**	**Fitted curve next mini-mum**	**Period bet-ween dates (yrs)**	**Refer Para.**
	Short cycle only	Long cycle only	**Both cycles**	Both cycles				
	Data discount per quarter							
	0.5%			0%				
BONDS (Data from30 June 1950 to **30 June 2017**)								
26.75 + 64.25	0.2	79.6	**93.2**	92.9	Dec 64	Mar 22	57.25	5.9, 5.10
63.0	(not applicable)		**79.6**	83.2	Sep 53	Sep 16	63.0	5.12
BONDS (Data from 30 June 1950 to **30 June 2021**)								
28.25 +72.0	2.8	81.1	**94.0**	92.8	Dec 63	Sep 05	41.75	5.9, 5.10
73.5	(not applicable)		**81.1**	77.8	Mar 48	Sep 21	73.75	5.15
CPI (Data from year-ending 30 September 1949 to **30 June 2021**)								
31.5 + 84.5	12.3	30.7	**57.0**	48.7	Jun 62	Jun 98	36.0	5.13
(next minima)					Jun 98	Mar 26	27.75	5.13
83.25	(not applicable)		**30.7**	17.7	Dec 34	Mar 18	83.25	5.15
AWOTE (Data from year-ending 30 September 1949 to **30 June 2021**)								
31.75 + 94.25	11.3	33.2	**54.6**	48.2	Mar 61	Sep 96	35.5	5.14
(next minima)					Sep 96	Mar 25	28.5	5.14
27.25	(not applicable)		**20.2**	24.1	Dec 90	Mar 18	27.25	5.14
AWOTE (Data from year-ending **30 September 1982** to **30 June 2021**)								
21.25	(not applicable)		**36.0**	35.0	Dec 95	Mar 17	21.25	5.14

For each sector the optimum result is illustrated in the charts on the following pages - the single cycles for Bonds and CPI are illustrated together in Figure 5.4.

5.9 Comparing the 2021 Bond results with the 2017 Bond results shows that both the short and long cycles have increased – as a consequence of the continued fall in Bond rates over the past 4 years. Comparing the first and third Bond fitted curves shows that the explained percentage of the sum of squared residuals has increased slightly (from 93.2 to 94.0). For the third fitted curve, a minimum is now evident at December 1963 and another at September 2005. However, it should be noted that, rounded to two-decimal places, the fitted curve from December 2004 to March 2007 is 5.28% for 11 consecutive quarters. From then the fitted curve falls steadily to a much more pronounced minimum (with an annual force of -0.2%) at December 2024.

5.10 For Bonds, the (28.25 + 72) year curve explained a very high 94.0% of the sum of squared residuals from the mean. However, the curve is a strange shape with flat values near the first and second minima. Figure 5.1 below graphs this curve and the actual data. It also includes the corresponding June 2017 curve (in orange) from Grenfell and Sneddon (2017), which is the first curve in Table 5.1 above. Because it is based on an extra four years of data the purple curve always differs from the orange curve (except where the two curves cross), however after September 2020 the difference exceeds 0.3% and becomes evident on the graph.

Figure 5.1 Ten-year bonds – economic cycles

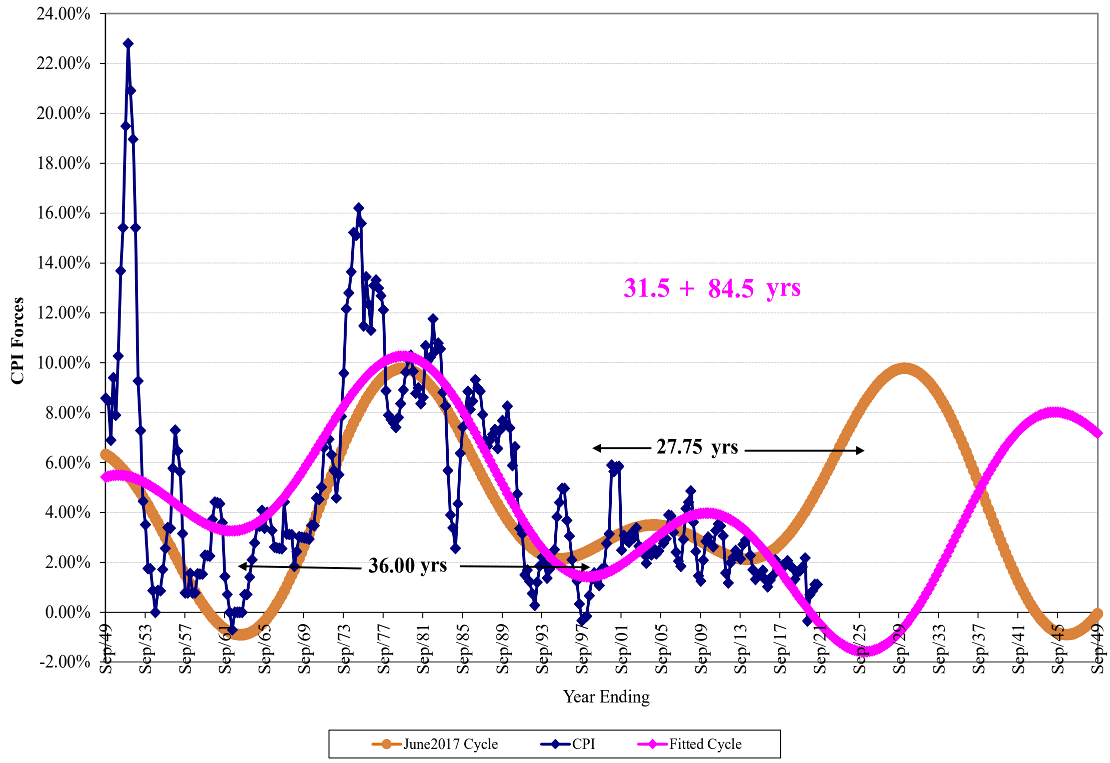


5.11 In Grenfell (2013) a (26.5 + 53) year curve for Bonds was mentioned. The curve, by design, had a long cycle twice the length of its short cycle and was therefore symmetrical in shape. However, the curve is not optimum and has not been considered here because the optimum long cycle is now about 2.5 times the short cycle.

5.12 Section 5.1 above explained that "The length of **past** economic cycles is a key determinate for setting the periods over which statistics are calculated for the purposes of "Step 2" in section 2.3". However, it is now evident that the sum of two sine curves technique, by itself, is not always suitable for determining the length of past economic cycles for this purpose. This occurs because the combined cycles in sections 5.9 and 5.10 above are now flat at or near the two minima and are "strange" or distorted shapes. The sum of two sine curves technique remains suitable for the purposes (1) to (4) in section 5.1 but something more is needed to satisfy "Step 2" in section 2.3. Thus, the single sine curves were included in section 5.8 and are commented on further in sections 5.14 and 5.15 below. By design the single sine curves do not give flat results or have distorted shapes.

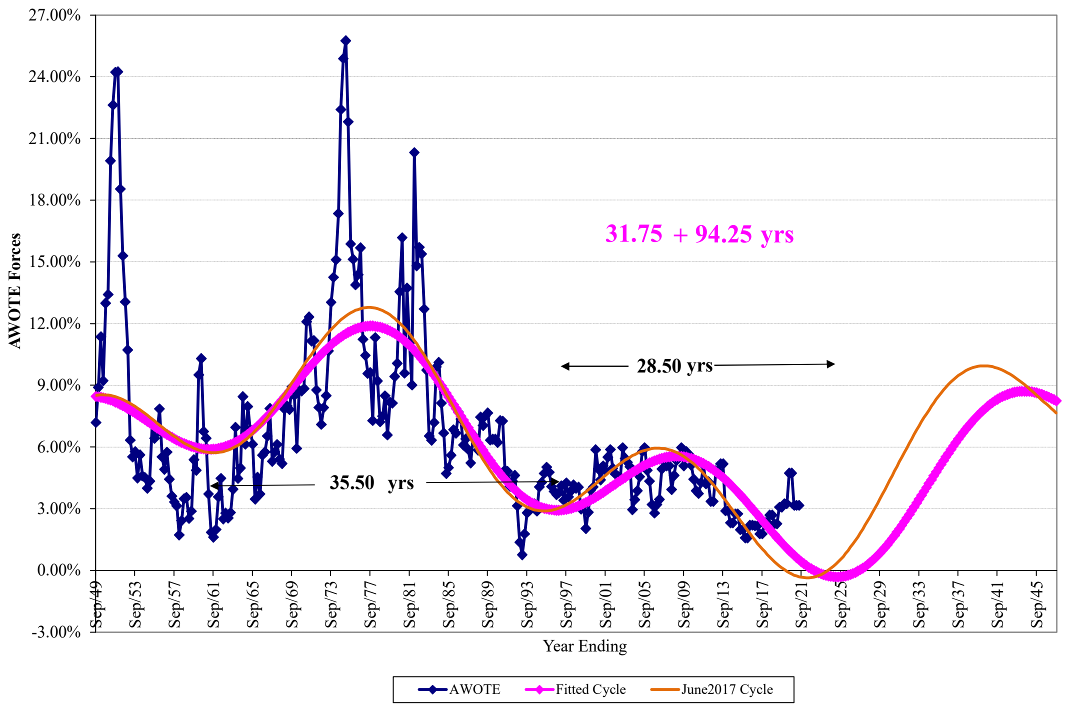
5.13 For CPI, the (31.5 + 84.25) year curve explained 57.0% of the sum of squared residuals. This was the highest percentage explained of any one curve or two-curve combination. The CPI curve-fitting is, significantly, not as close to the data as for Bonds and is more period dependent. The fitted curve now has quite a different shape to the orange curve relating to Grenfell and Sneddon (2017). For example, from June 2014 to March 2026 the orange curve rises sharply but during this period the purple curve falls sharply. Figure 5.2 below graphs both curves and the actual data.

Figure 5.2 CPI – economic cycles



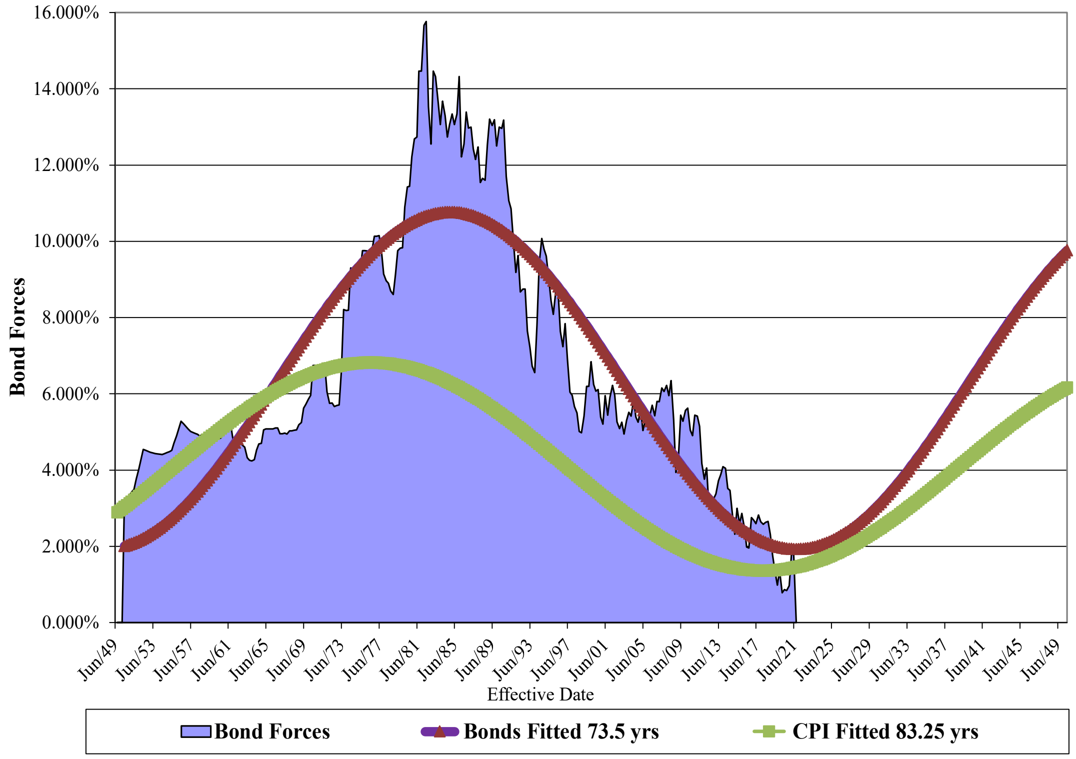
5.14 For AWOTE, the (31.75 + 94.25) year curve explained 54.6% of the sum of squared residuals. The AWOTE curve-fitting is also, significantly, not as close to the data as for Bonds. Figure 5.3 below graphs this curve and the actual data. For AWOTE, the 27.25-year single curve (which is not illustrated in a chart) only explained 20.2% of the sum of squared residuals. As explained in section 4.1, prior to 30 September 1981 the W sector data was not based on "full time adults". If this more volatile data is disregarded, the single cycle optimum result is a 21.25-year curve and the sum of squared residuals improves to 36.0%.

Figure 5.3 AWOTE – economic cycles



5.15 Figure 5.4 graphs the single cycle fitted curves for Bonds and CPI and the actual data for Bonds. For the 73.5-year fitted Bonds curve the minimum is reached in September 2021 but for the 83.25-year CPI curve the minimum was reached three and a half years earlier, in March 2018. This CPI lead, and the similar AWOTE lead, are analysed in more detail in Section 14.

Figure 5.4 Bonds and CPI – single cycles



5.16 In Grenfell (2013) it was mentioned that a (4.222 + 9.066) year curve for Shares explained an optimal 20.9% of the sum of squared residuals from the mean. The 4.222-year and 9.066-year periods were equal to 16 quarters plus 81 days and 36 quarters plus 24 days respectively. In Grenfell and Sneddon (2017) the optimal curve for Shares was a (3.381 + 8.980) year curve and this explained 20.4% of the sum of squared residuals from the mean. Based on the latest data the optimal curve for Shares is now (4.189 + 8.964) years, which is similar to the Grenfell (2013) results, but the explained percentage of the sum of squared residuals has fallen to 18.4%. Since the explained percentage of the sum of squared residuals has fallen below 20%, this curve has now not been included in Table 5.1.

5.17 In Grenfell (2007 to 2017) it was mentioned that the approximately 4-year cycles evident in the Share curves could perhaps be attributable to United States of America Presidential elections. This possibility is analysed in Nickles (2004). This shows that during the period 1942 to 2002 USA stock market cycles averaged 4.02 years. During this period, bull markets averaged about three years, while bear markets averaged less than a year.

5.18 Consideration of the second last column of Table 5.1 above shows that the periods between **past** minima have ranged from 21.25 to 83.25 years depending on the curve-fitting and whether one's focus is on Bonds, CPI or AWOTE. Much of the work in the following sections is based on 46-year past periods because:

(a) 44-year periods were used in Grenfell (2013) and Grenfell and Sneddon (2017) but the various short, long and single cycle lengths have **increased** since 2017 except for the single AWOTE curve, but

(b) for CPI and AWOTE and, to a lesser extent Bonds, the two-cycle charts indicate that, in the post 1985 period, much **shorter** curves are starting to emerge (perhaps due to the increased capacity and wider availability of computing for financial business and the ability to more quickly respond to new data), and

(c) the following Summary suggests an overall average result of approximately 46 years.

SUMMARY

	Single Cycle Charts		Double Cycle Charts	
	Years between Minima	Explained ppn. of Squared Residuals	Years between Minima	Explained ppn. of Squared Residuals
Bonds	73.50	81.13%	41.75	93.95%
CPI	83.25	30.72%	27.75	56.96%
AWOTE >Sep 48	27.25	20.15%	28.50	54.61%
AWOTE > Sep 81	21.25	36.00%		
Balanced	45.00	77.40%	45.00	77.40%
Cap Stable	46.50	90.05%	46.50	90.05%
Weighted av. (years) (red font 50%)	**54.81**		**39.49**	
Average weight (ex Balanced + Cap Stable)		46.64%		68.51%
	Weighted av. (Years)		**45.70**	
	Rounded		**46 years**	

The above Summary includes some results from the analysis in sections 9.10 and 9.11. That analysis shows that the past period between minima (denoted "Balanced" in the Summary) for the Balanced portfolio log "discounted unit price" trend line was 45.0 years and the explained proportion of squared residuals was 77.4%, The past period between minima (denoted "Cap Stable)" in the Summary) for the Capital Stable portfolio log "discounted unit price" trend line was 46.5 years.

5.19 However, for some sectors (e.g. those denoted G, H, J, L, and N) the 46 years chosen in section 5.18 will often extend back into the "back-dated data" periods described in Section 4. Thus, it was considered desirable, for comparison, to also consider a shorter period, at least for risk margins and coefficients of variation. A period of 28 years was chosen for this purpose based on the three green-shaded results in the above Summary.

5.20 A further reason for adopting the shorter 28-year period, was that feedback after presentation of Grenfell (2017) indicated some concern about possible over-reliance on data extending back over 44 years.

5.21 **It is considered desirable for the setting of long-term assumptions to always analyse past historical results over a full economic cycle** (or in the more general sense, over <u>an integer number</u> of cycles). Thus 46 years was chosen for most calculations and 28 years was chosen for the second alternative set of risk margin and coefficient of variation calculations. The importance of using at least one full economic cycle is supported by Bernstein (1997) who "extracted the long-run rate of return on long-term bonds by searching for widely separated dates where yields are identical … and then calculated the annualised growth in the bond relative … between those dates".

6 Risk Margins

6.1 For the purposes of this paper, risk margins are calculated relative to the ten-year bond rate. The risk margin is the excess of the sector annual investment return over the annualised effective point-in-time bond rate (but see paragraph 6.4).

6.2 For sector D (10-year bonds) the point-in-time bond rate is taken at mid-year, that is 6 months prior to the end of the period over which the annual investment returns are determined (but see paragraph 6.4).

6.3 Now consider the correlation between the F (Fixed Interest) sector forces and D sector forces for various lags:

Table 6.1 D and F sector correlations

24 yr Correlation of F against D lagged by Y years:									
Y	0	0.25	0.5	0.75	1	1.25	1.5	1.75	2
End date									
30/06/1984	.333	.402	.462	.477	.482	.484	.461	.435	.402
30/06/1985	.370	.435	.492	.507	.511	.510	.490	.466	.432
30/06/1986	.416	.474	.528	.541	.5414	.5410	.525	.500	.468
30/06/1987	.468	.524	.580	.588	.589	.588	.570	.544	.515
30/06/1988	.487	.544	.602	.609	.608	.611	.594	.571	.545
30/06/1989	.477	.538	.597	.603	.599	.602	.581	.555	.528
30/06/1990	.493	.555	.616	.620	.615	.615	.595	.570	.542
30/06/1991	.519	.585	.645	.648	.642	.644	.622	.594	.562
30/06/1992	.513	.593	.667	.6775	.675	.6767	.655	.628	.596
30/06/1993	.452	.539	.627	.643	.643	.650	.628	.598	.566
30/06/1994	.393	.489	.588	.604	.604	.614	.591	.559	.527
30/06/1995	.441	.552	.655	.666	.660	.669	.650	.618	.584
30/06/1996	.514	.630	.735	.739	.718	.708	.673	.642	.616
30/06/1997	.362	.487	.629	.643	.625	.629	.605	.583	.556
30/06/1998	.274	.405	.539	.545	.521	.523	.498	.476	.455
30/06/1999	.295	.412	.534	.542	.521	.521	.496	.474	.453
30/06/2000	.356	.468	.578	.584	.563	.561	.539	.515	.491
30/06/2001	.366	.470	.577	.584	.562	.556	.527	.501	.481
30/06/2002	.414	.511	.610	.613	.593	.592	.565	.540	.520
30/06/2003	.430	.522	.6172	.6169	.597	.598	.576	.556	.542
30/06/2004	.500	.580	.664	.656	.631	.625	.599	.580	.568
30/06/2005	.586	.648	.723	.707	.671	.658	.627	.602	.586
30/06/2006	.593	.657	.728	.713	.680	.667	.640	.619	.605
30/06/2007	.597	.660	.732	.723	.691	.675	.647	.625	.612
30/06/2008	.609	.676	.749	.742	.714	.699	.666	.645	.640
30/06/2009	.618	.689	.766	.757	.730	.711	.676	.656	.649
30/06/2010	.603	.684	.760	.753	.718	.700	.664	.644	.636
30/06/2011	.561	.653	.735	.729	.694	.666	.629	.608	.601
30/06/2012	.539	.643	.736	.733	.693	.664	.629	.605	.590
30/06/2013	.596	.692	.780	.771	.736	.707	.667	.644	.630
30/06/2014	.561	.676	.778	.765	.718	.681	.635	.613	.599
30/06/2015	.348	.500	.637	.613	.550	.504	.444	.415	.407
30/06/2016	.254	.417	.559	.531	.456	.386	.310	.282	.274
30/06/2017	.251	.405	.540	.517	.430	.355	.274	.230	.210
30/06/2018	.527	.618	.723	.702	.647	.602	.545	.518	.511
30/06/2019	.388	.495	.619	.593	.554	.539	.497	.467	.437
30/06/2020	.225	.347	.496	.480	.435	.416	.366	.333	.317
30/06/2021	.209	.317	.449	.424	.378	.353	.286	.233	.189

6.4 Table 6.1 shows that when D sector forces are lagged by say a further 9 months (giving a total lag of 15 months) the correlation between D and F sector forces increases to a maximum, or close to the maximum. For the last 19 years the maximum occurs in the 6 month column but the difference between the 6 month lag correlations and the 9 month ones are all less than 0.030. Except for five results prior to 1996, the maximum tabulated correlation between D and F sector forces has always occurred between:

- 6 months (giving a total lag of 12 months), and
- 12 months (giving a total lag of 18 months).

There is evidence that the lag shortened from 1993 to about 2003, probably primarily due to a contracting F sector average maturity "duration". What it might do in the future is uncertain, however for the specific purposes of Steps 3 to 5 of the methodology, past experience is relevant, not future predictions. Therefore, to give greater stability to F sector risk margins, and to a lesser extent other sector risk margins, **all the following risk margins are defined as**:

(annual investment return) less (bond rate lagged 15 months)

6.5 The choice in the previous paragraph of a further 9 months (giving a total of 15 months) has no significant effect on the general level of the following margins, other than introducing some stability to some of the results. This greater stability of the risk margins improves the accuracy of the quadratic trend fitting and the two-year projections in steps 5 and 6 of the methodology (see Section 2). The 9-month adjustment has no practical impact on the application of the resultant margins or on their use in stochastic or other models.

6.6 Once assumptions have been set for risk margins (for the 14 sectors other than bonds) and for the mean bond rate, assumptions for means for the 14 sectors can be determined from the formula:

Arithmetic mean rate = risk margin + mean bond rate

6.7 Figure 6.1 and Table 6.4 show the risk margin results. For the D sector (bonds) the means, after lagging the bond rate by 15 months, are shown.

EXPLANATION

6.8 In the following figures and tables, the meaning of "Period" is:

Table 6.2 Statistics for each "period"

Period	Average statistic of four periods ending:
15	30/9/05, 31/12/05, 31/3/06 and 30/6/06
14	30/9/06, 31/12/06, 31/3/07 and 30/6/07
13	30/9/07, 31/12/07, 31/3/08 and 30/6/08
12	30/9/08, 31/12/08, 31/3/09 and 30/6/09
11	30/9/09, 31/12/09, 31/3/10 and 30/6/10
10	30/9/10, 31/12/10, 31/3/11 and 30/6/11
9	30/9/11, 31/12/11, 31/3/12 and 30/6/12
8	30/9/12, 31/12/12, 31/3/13 and 30/6/13
7	30/9/13, 31/12/13, 31/3/14 and 30/6/14
6	30/9/14, 31/12/14, 31/3/15 and 30/6/15
5	30/9/15, 31/12/15, 31/3/16 and 30/6/16
4	30/9/16, 31/12/16, 31/3/17 and 30/6/17

3	30/9/17, 31/12/17, 31/3/18 and 30/6/18
2	30/9/18, 31/12/18, 31/3/19 and 30/6/19
1	30/9/19, 31/12/19, 31/3/20 and 30/6/20
0	30/9/20, 31/12/20, 31/3/21 and 30/6/21
0	*30/9/20, 31/12/20, 31/3/21 and 30/6/21 (trend)*
-1	*30/9/21, 31/12/21, 31/3/22 and 30/6/22 (projections)*
-2	*30/9/22, 31/12/22, 31/3/23 and 30/6/23 (projections)*
-3	*30/9/23, 31/12/23, 31/3/24 and 30/6/24 (projections)*
-4	*30/9/24, 31/12/24, 31/3/25 and 30/6/25 (projections)*
-5 & -6	calculated as above, but not tabulated

6.9 The above trends and 'projections' (or extrapolations) are based on weighted quadratic EXCEL trend functions (refer section 2.7). The "period" is indicated on the X-axis of figures in Sections 6, 7, 8, 11 and 12.

6.10 The following codes are used:

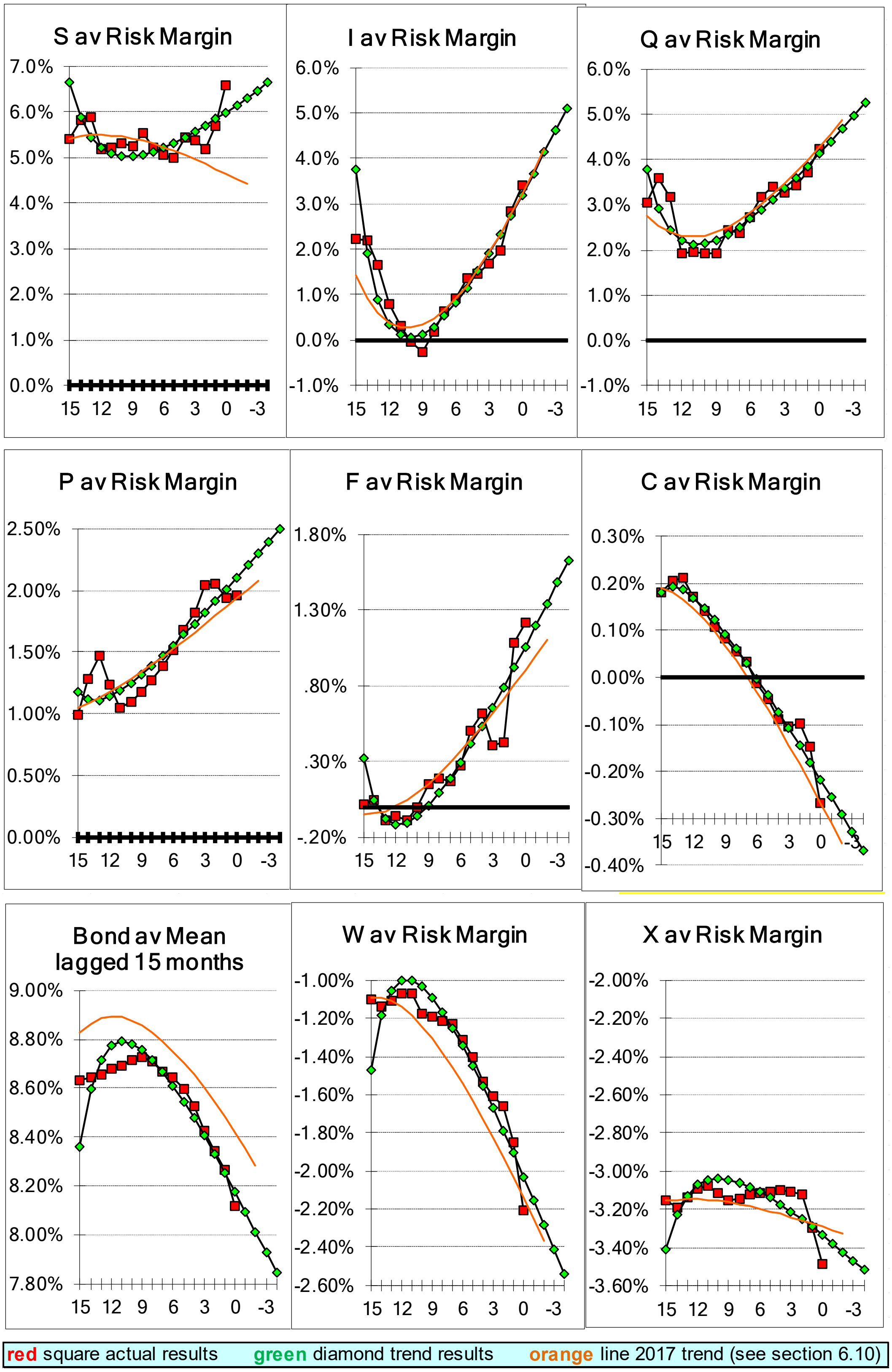
Sector	Relates to
D	ten-year bond rate in the middle of the year (i.e. lagged by 6 months)
D-9	ten-year bond rate lagged by 15 months (i.e. as for D but lagged by a further 9 months)
X	annual increase in the Consumer Price Index
W	annual increase in AWOTE
[square]	actual results based on 46-year averages
[diamond]	trend results based on 46-year averages
[orange line]	trend or projections from Grenfell and Sneddon (2017) that were based on 44-year averages
italics	*trend results or projections*

6.11 Taking into consideration the following three "pointers", the "NEW" assumptions are set (refer section 2.8) for risk margins and coefficients of variation:

Table 6.3 "Pointers"

Pointer	Refers to:
46 –2	Average 46-year projected statistic for "Period -2"
28 –2	Average 28-year projected statistic for "Period -2"
OLD	Previous assumptions based on data up to 30/6/2017

Figure 6.1 Risk margins over 46 years



X-axis: Period 15 = average 46 years ending 30/9/05, 31/12/05, 31/3/06 and 30/6/06
Period 0 = average 46 years ending 30/9/20, 31/12/20, 31/3/21 and 30/6/21 (see sect. 6.8)

Table 6.4 Risk margins over 46 years
Actual and *Quadratic Trend*

Period	S	I	Q	P	L	H	F	G
15	5.4%	2.2%	3.0%	1.0%	1.3%	4.0%	0.0%	0.9%
14	5.8%	2.2%	3.6%	1.3%	1.4%	4.1%	0.0%	0.9%
13	5.9%	1.7%	3.2%	1.5%	1.3%	3.7%	-0.1%	0.9%
12	5.2%	0.8%	1.9%	1.2%	1.3%	2.9%	-0.1%	1.0%
11	5.2%	0.3%	1.9%	1.0%	1.3%	2.7%	-0.1%	0.9%
10	5.3%	0.0%	1.9%	1.1%	1.3%	2.6%	0.0%	0.9%
9	5.3%	-0.3%	1.9%	1.2%	1.4%	2.4%	0.1%	0.9%
8	5.5%	0.2%	2.4%	1.3%	1.4%	2.9%	0.2%	0.9%
7	5.2%	0.6%	2.4%	1.4%	1.5%	3.1%	0.2%	0.9%
6	5.0%	0.9%	2.7%	1.5%	1.5%	3.1%	0.3%	0.9%
5	5.0%	1.4%	3.2%	1.7%	1.5%	3.4%	0.5%	0.9%
4	5.4%	1.5%	3.4%	1.8%	1.5%	3.6%	0.6%	0.9%
3	5.4%	1.7%	3.3%	2.0%	1.5%	3.7%	0.4%	0.8%
2	5.2%	2.0%	3.4%	2.1%	1.6%	3.5%	0.4%	0.8%
1	5.7%	2.8%	3.7%	1.9%	1.7%	4.1%	1.1%	1.0%
0	6.6%	3.4%	4.2%	2.0%	1.7%	5.0%	1.2%	0.9%
0	*6.0%*	*3.2%*	*4.1%*	*2.1%*	*1.7%*	*4.6%*	*1.1%*	*0.9%*
-1	*6.1%*	*3.7%*	*4.4%*	*2.2%*	*1.7%*	*4.9%*	*1.2%*	*0.9%*
-2	*6.3%*	*4.1%*	*4.7%*	*2.3%*	*1.8%*	*5.2%*	*1.3%*	*0.9%*
-3	*6.5%*	*4.6%*	*5.0%*	*2.4%*	*1.8%*	*5.5%*	*1.5%*	*0.9%*
-4	*6.6%*	*5.1%*	*5.2%*	*2.5%*	*1.9%*	*5.8%*	*1.6%*	*0.9%*

J	C	N	B	D-9	W	X	Period
-0.2%	0.2%	1.1%	0.2%	8.63%	-1.1%	-3.2%	15
-0.2%	0.2%	1.1%	0.2%	8.64%	-1.1%	-3.2%	14
-0.2%	0.2%	1.0%	0.3%	8.66%	-1.1%	-3.1%	13
-0.3%	0.2%	1.0%	0.3%	8.68%	-1.1%	-3.1%	12
-0.2%	0.1%	1.0%	0.2%	8.69%	-1.1%	-3.1%	11
-0.1%	0.1%	1.1%	0.2%	8.72%	-1.2%	-3.1%	10
0.0%	0.1%	1.3%	0.2%	8.73%	-1.2%	-3.2%	9
0.1%	0.1%	1.4%	0.1%	8.71%	-1.2%	-3.1%	8
0.1%	0.0%	1.3%	0.1%	8.67%	-1.2%	-3.1%	7
0.2%	0.0%	1.5%	0.1%	8.64%	-1.3%	-3.1%	6
0.4%	0.0%	1.7%	0.0%	8.59%	-1.4%	-3.1%	5
0.5%	-0.1%	1.7%	0.0%	8.53%	-1.5%	-3.1%	4
0.4%	-0.1%	1.5%	0.0%	8.43%	-1.6%	-3.1%	3
0.4%	-0.1%	1.5%	0.0%	8.34%	-1.7%	-3.1%	2
0.9%	-0.1%	1.9%	-0.1%	8.27%	-1.9%	-3.3%	1
1.0%	-0.3%	1.9%	-0.2%	8.11%	-2.2%	-3.5%	0
0.9%	*-0.2%*	*1.8%*	*-0.2%*	*8.18%*	*-2.0%*	*-3.3%*	*0*
1.0%	*-0.3%*	*1.9%*	*-0.2%*	*8.10%*	*-2.2%*	*-3.4%*	*-1*
1.2%	*-0.3%*	*2.0%*	*-0.3%*	*8.01%*	*-2.3%*	*-3.4%*	*-2*
1.3%	*-0.3%*	*2.0%*	*-0.3%*	*7.93%*	*-2.4%*	*-3.5%*	*-3*
1.4%	*-0.4%*	*2.1%*	*-0.3%*	*7.84%*	*-2.5%*	*-3.5%*	*-4*

Pointers (risk margins)

	S	I	Q	P	L	H	F	G	J	C	N	B	D-9	W	X
46 -2	6.3%	4.1%	4.7%	2.3%	1.8%	5.2%	1.3%	0.9%	1.2%	-0.3%	2.0%	-0.3%	8.01%	-2.28%	-3.42%
28 -2	4.7%	3.3%	5.3%	3.0%	1.0%	5.2%	1.2%	-0.4%	1.4%	-1.0%	1.5%	-1.1%	4.57%	-0.99%	-2.64%
OLD	4.4%	3.0%	3.0%	1.8%	1.6%	3.1%	0.7%	0.7%	0.5%	-0.2%	1.5%	-0.2%	5.50%	-1.80%	-3.00%
NEW	5.1%	4.1%	4.1%	2.3%	1.7%	4.3%	1.0%	0.9%	0.9%	-0.3%	1.5%	-0.3%	5.00%	-2.00%	-3.00%

6.12 In all cases the "NEW" risk margin assumptions are set in the range from:

a) the lowest of the first 3 pointers, to
b) the highest of the first 3 pointers.

However, in many cases this is a wide range. Considerable weight was given to the "OLD" (see section 6.11) risk margin assumptions. The following further brief explanations should give some indication of the setting process:

S (shares) Set equal to the average of all 3 pointers (=5.14%) which was rounded to 5.1%.

I (int'l shares) Combined with Q because of the divergent results – for many years the I sector has had significantly lower risk margins and significantly higher CoV's than the Q sector, but a study of Tables 6.4 and 7.1 shows that these divergent results are steadily diminishing. Both I and Q risk margins were set equal to "46 -2" for I = 4.1% (which equals the "OLD" pointers plus 1.1%). The average risk margins for both I and Q have steadily increased each year since their low points about 10 years ago.

Q (prop trust) Combined with I (because of the divergent results). The 'combined' assumption was 4.1%. It was noted that this falls 64% of the way between the "NEW" P and S risk margins.

P (direct prop) Set equal to "46 –2" (= 2.3%). The average of "OLD" and "28 -2" is also approximately 2.3% ((2.96%+1.80%)/2 =2.38%).

H (hedged i.s.) Set equal to 4.3%, which is 58% of the way between "OLD" and "46 -2". This doubles the 0.1% differential between hedged and unhedged international shares assumed in the "OLD" assumptions (which again implies an assumed slight long-term upward trend in the Australian dollar with our economy running marginally stronger than the US and Europe).

F (fixed int) Set equal to the average of "OLD" and "46 –2" ((0.70%+1.34%)/2 = 1.02%) which was rounded to 1.0%. This is slightly less than "28 -2" which is 1.2%.

G (govt semi) Set equal to “46 –2” (= 0.9%). This is now 0.1% less than the risk margin for F (previously equal to the risk margin for F).

J (int’l fxd int) Set equal to F less 0.1%, partly to cover the cost of hedging (including the direct transaction costs and indirect transaction costs arising from the re-investment of cash flows from expiry of forward foreign exchange contracts). Also noted that the difference between F and J was 0.18% for “46 –2”.

C (cash) Set equal to “46 –2” (= -0.3%) which is also the B risk margin.

N (infln linked) Set equal to “28 -2” (= 1.53%, rounded down to 1.5%). This is 0.5% more than the risk margin for F. Because of the short data term (30.25 years) the results for “46 -2” were disregarded. Noted that for the 28-year trend values, N-F slowly and steadily trends towards 0.5% at Periods -5 and -6.

D (bonds) The base was set equal to 5.0% per annum. This is 0.5% lower than “OLD”. This drop is consistent with the fall in “46 -2”. The long-term assumption is higher than current effective 10-year government bond rates (e.g. 0.88% effective at 30 June 2020 and 1.81% at 31 March 2021) but it was noted that 40-year simulations with the “NEW” D sector means, CoV, skewness and kurtosis, result in Bond annual rates typically (i.e. based on median lows and median highs) ranging from about 1.1%% to 9.4%.

X (CPI) Set equal to the average of “46 –2” and “28 -2” (-(3.42%+2,64%)/2 = -3.03%) rounded up to -3.0% which corresponds to a mean annual inflation rate of 2.0% per cent. The latter is equal to the bottom of the Reserve Bank of Australia target band of 2% to 3%.

W (AWOTE) Set equal to the average of “OLD” and “46 –2” (-(2.28%-+1.80%)/2 = -2.04%) which was rounded to -2.0%. The “productivity” difference between W and X for “46 –2” was 1.14%. However, the “productivity” difference between W and X for “NEW” was set equal to 1.0% which is consistent with the 46-year trend values of W-X which are slowly and steadily falling towards 1.0% at Period -4.

7 Coefficients of Variation

7.1 The coefficient of variation is equal to the standard deviation divided by the mean. Figure 7.1 and Table 7.1 show the results.

7.2 Once assumptions have been set for risk margins, the mean bond rate and, for coefficients of variation, assumptions for standard deviations for the 15 sectors can be determined from the formula:

Standard deviation = (risk margin + mean bond rate) * (coefficient of variation)

7.3 Whether standard deviation is a reasonable surrogate for risk is an open question reaching far beyond the scope of this paper. However, bear in mind the quotation from Waite (2009) at the bottom of this page.

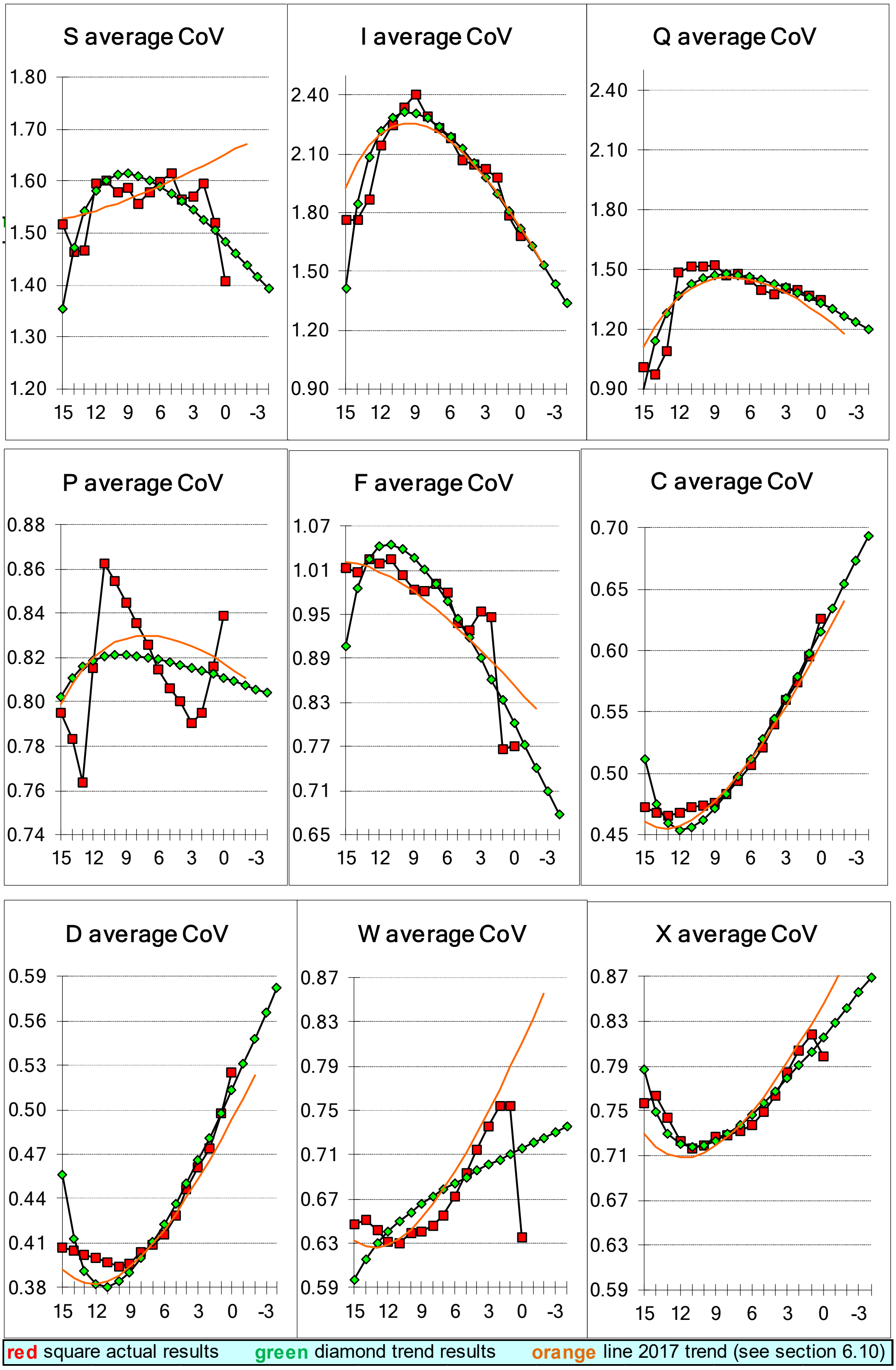
7.4 Brown (2013) refers to the "index of stability" or "ios" which is the reciprocal of the coefficient of variation. In the engineering literature, it is known as the "signal to noise ratio". In the application to investments we can assume that the mean return is positive. The standard deviation is always positive; hence it follows that the ios for investment returns will always be positive. Brown (2013) tabulates the following values of ios based on investment forces of return, before tax and fees, using the *Austmod* historical database for the period from September 1965 to September 2012:

I (int'l shares, unhedged)	0.33
H (hedged int'l shares)	0.48
S (shares, Australian)	0.52
Q (property trusts)	0.55
F (fixed interest)	0.98
P (direct property)	1.07
J (int'l fixed interest)	1.19
N (inflation linked)	1.19
G (semi-govt, 0-3yrs)	1.79
L (loans/credit)	2.20
C (cash)	2.20

REMEMBER

> *"There is an acceptable four letter word that is at the heart of the current financial crisis. It is "risk". Risk is at the centre of the actuarial profession and the area of ERM (Enterprise Risk Management) is gaining new ground in the actuarial world."*
>
> *Waite (2009)*

Figure 7.1 Coefficients of variation over 46 years



X-axis: Period 15 = average 46 years ending 30/9/05, 31/12/05, 31/3/06 and 30/6/06
Period 0 = average 46 years ending 30/9/20, 31/12/20, 31/3/21 and 30/6/21 (see section 6.8)

Table 7.1 Coefficients of variation over 46 years

Actual and *Quadratic Trend*

Period	S	I	Q	P	L	H	F	G
15	1.52	1.77	1.01	0.79	0.50	1.46	1.01	0.59
14	1.46	1.76	0.97	0.78	0.49	1.45	1.01	0.59
13	1.46	1.87	1.09	0.76	0.49	1.49	1.02	0.59
12	1.60	2.14	1.48	0.82	0.50	1.67	1.02	0.58
11	1.60	2.25	1.52	0.86	0.50	1.70	1.02	0.59
10	1.58	2.34	1.51	0.85	0.49	1.71	1.00	0.59
9	1.59	2.41	1.52	0.84	0.48	1.75	0.98	0.58
8	1.56	2.29	1.47	0.84	0.48	1.68	0.98	0.59
7	1.58	2.23	1.48	0.83	0.48	1.66	0.99	0.60
6	1.60	2.18	1.45	0.81	0.48	1.65	0.98	0.61
5	1.62	2.07	1.40	0.81	0.48	1.60	0.94	0.61
4	1.56	2.05	1.38	0.80	0.49	1.58	0.93	0.62
3	1.57	2.02	1.40	0.79	0.51	1.58	0.95	0.64
2	1.59	1.98	1.40	0.79	0.51	1.61	0.95	0.65
1	1.52	1.78	1.37	0.82	0.50	1.52	0.77	0.62
0	1.41	1.68	1.34	0.84	0.51	1.39	0.77	0.65
0	*1.48*	*1.72*	*1.33*	*0.81*	*0.51*	*1.45*	*0.80*	*0.65*
-1	*1.46*	*1.63*	*1.30*	*0.81*	*0.51*	*1.41*	*0.77*	*0.66*
-2	*1.44*	*1.53*	*1.27*	*0.81*	*0.52*	*1.37*	*0.74*	*0.67*
-3	*1.42*	*1.44*	*1.24*	*0.81*	*0.52*	*1.33*	*0.71*	*0.67*
-4	*1.39*	*1.34*	*1.20*	*0.80*	*0.53*	*1.28*	*0.68*	*0.68*

J	C	N	B	D	W	X	Period
0.85	0.47	0.81	0.50	0.41	0.65	0.76	15
0.84	0.47	0.81	0.50	0.41	0.65	0.76	14
0.84	0.47	0.81	0.49	0.40	0.64	0.74	13
0.86	0.47	0.81	0.49	0.40	0.63	0.72	12
0.85	0.47	0.82	0.49	0.40	0.63	0.72	11
0.83	0.47	0.80	0.50	0.39	0.64	0.72	10
0.81	0.48	0.79	0.50	0.40	0.64	0.73	9
0.81	0.48	0.79	0.51	0.40	0.65	0.73	8
0.81	0.49	0.81	0.52	0.41	0.66	0.73	7
0.80	0.51	0.79	0.53	0.42	0.67	0.74	6
0.77	0.52	0.77	0.54	0.43	0.69	0.75	5
0.76	0.54	0.78	0.56	0.45	0.71	0.76	4
0.78	0.56	0.80	0.58	0.46	0.74	0.78	3
0.78	0.57	0.80	0.60	0.47	0.75	0.80	2
0.65	0.60	0.73	0.62	0.50	0.75	0.82	1
0.65	0.63	0.74	0.65	0.53	0.63	0.80	0
0.67	*0.62*	*0.75*	*0.64*	*0.51*	*0.72*	*0.82*	*0*
0.65	*0.63*	*0.74*	*0.66*	*0.53*	*0.72*	*0.83*	*-1*
0.63	*0.65*	*0.73*	*0.68*	*0.55*	*0.73*	*0.84*	*-2*
0.61	*0.67*	*0.72*	*0.69*	*0.57*	*0.73*	*0.86*	*-3*
0.58	*0.69*	*0.72*	*0.71*	*0.58*	*0.74*	*0.87*	*-4*

Pointers (CoV)

	S	I	Q	P	L	H	F	G
46 -2	1.44	1.53	1.27	0.81	0.52	1.37	0.74	0.67
28 -2	1.46	2.16	1.78	0.77	0.25	1.62	0.78	0.51
OLD	1.616	1.623	1.624	0.877	0.493	1.558	0.758	0.613
NEW	1.535	1.539	1.539	0.822	0.523	1.398	0.750	0.644

J	C	N	B	D	W	X	
0.63	0.65	0.73	0.68	0.55	0.73	0.84	46 -2
0.64	0.38	0.91	0.38	0.41	0.33	0.48	28 -2
0.633	0.566	0.714	0.585	0.472	0.758	0.800	OLD
0.627	0.618	0.830	0.639	0.480	0.700	0.800	NEW

7.5 In all cases the "NEW" coefficient of variation assumptions are set in the range from:

a) the lowest of the first 3 pointers, to
b) the highest of the first 3 pointers.

Considerable weight was given to the "OLD" (see section 6.11) coefficient of variation assumptions. The following brief explanations should give some indication of the setting process:

S (shares) Set equal to the average of "46 –2" and "OLD" and rounded up to 1.535, which gives a standard deviation of 15.50% (equal to "OLD" – 0.50%).

I (int'l shares) Combined with Q (as for risk margins). Hence both CoVs were set equal to "46 -2" for I =1.53 which was rounded up to 1.539 so that the standard deviation (14.0%) is an integral multiple of 0.1%.

Q (prop trust) Combined with I (as for risk margins). The 'combined' assumption was 1.539 which gives a standard deviation (14.0%) that is 0.2% higher than "OLD".

P (direct prop) Set equal to the maximum of "46 –2" and "28 –2" (=0.81) which was rounded up to 0.822 to give a standard deviation (6.00%) which is an integral multiple of 0.1%.

H (hedged i.s.) Set equal to "46 –2" (=1.37), rounded to 1.398 so that the standard deviation (13.00%) is an integral multiple of 0.1%.

F (fixed int) Because of the shorter F sector average maturity "duration" noted in section 6.4, the 46-year average results were given less weight. The three pointers are all quite close. The average of "46 –2" and "28 –2" was 0.759. Hence the "NEW" value was set equal to this and rounded to 0.750 to give a standard deviation of 4.50%.

J (int'l fxd int) Also set equal to the average of "46 -2" and "28 –2" (and rounded to 0.627 to give a standard deviation of 3.70%. The three pointers are all very close.

C (cash) Set equal to the average of "OLD' and "46 –2" (=0.610) and rounded up to 0.618. This resulted in a small decrease in the standard deviation from 3.0% (for "OLD") to 2.9% (for "NEW").

N (infln linked) As for risk margins, the results for “46 –2” were disregarded. The values for “28 –2” and “OLD” were 0.91 and 0.714 respectively. The average of these two values was chosen and rounded up to .830 so that the CoV and the standard deviation were reasonably consistent with those for F.

B (bills) Consistent with C, set equal to the average of “OLD’ and “46 –2”. This resulted in a small 0.10% decrease in the standard deviation.

D (bonds) Set equal to the average of “46 –2” and “28 –2” (=.481). After rounding to 0.480, this resulted in a small decrease in the standard deviation from 2.60% (for “OLD”) to 2.50% (for “NEW”).

W (AWOTE) Set equal to “46 –2” rounded down to 0.700 so that the standard deviation is an integral multiple of 0.1%. Noted that 40-year simulations, with the “NEW” W sector means, CoV, skewness and kurtosis, result in AWOTE annual rates typically (i.e. based on medians) ranging from about 1.1% to 10.0% reflecting in particular the very high positive skewness.

X (CPI) Consistent with W, set equal to “46 –2” and rounded down (to 0.800). This resulted in an unchanged CoV and a decrease in the standard deviation from 2.00% (for “OLD”) to 1.60% (for “NEW”).

8 Means and Standard Deviations

8.1 When formulating assumptions for stochastic models, risk margins and coefficients of variation from the previous two sections were used to determine corresponding means and standard deviations. The resulting assumptions are summarised in Section 16. In **this section** the 46-year and 28-year means and standard deviations are tabulated, **purely for historical interest.**

8.2 It should be noted that all the results in this section, and all results up to and including Section 16, are **gross of tax and gross of fees**.

8.3 Grenfell (1997) included results for a "Balanced" portfolio based on the following fixed asset allocations (which were derived from the Mercer Pooled Fund asset-weighted average benchmarks at **28 February 1997):**

Sectors	**S**	**I**	**Q**	**P**	**F**	**J**	**C**	**N**
Balanced	35%	20%	4%	7%	20%	6%	8%	0%

Grenfell (2009 to 2017) included results for "Balanced" and "Capital Stable" portfolios based on the following fixed asset allocations:

Sectors	**S**	**I**	**H**	**Q**	**P**	**F**	**J**	**C**	**N**
Balanced	36%	19%	6%	7%	2%	16%	7%	5%	2%
Capital Stable	15%	7%	2%	3%	2%	32%	12%	25%	2%

These asset allocations proportions were equal to the Mercer Pooled Fund asset-weighted average benchmarks at **30 September 2006** and at **30 June 2008** with some minor rounding. There were no significant changes between these two dates; the benchmarks have not been updated since 2008.

To bring the proportions closer to those published in APRA quarterly Superannuation Performance Statistics, the following slight changes have now been made to the above allocations:

(a) S sector reduced by 4%
(b) I sector increased by 3%
(c) H sector increased by 1%
(d) P sector increased by 1%
(e) C sector increased by 3% (only for Balanced)
(f) F sector reduced by 4% (for Balanced), 1% (for Capital Stable)

The resultant fixed asset allocations are now:

Sectors	**S**	**I**	**H**	**Q**	**P**	**F**	**J**	**C**	**N**
Balanced	32%	22%	7%	7%	3%	12%	7%	8%	2%
Capital Stable	11%	10%	3%	3%	3%	31%	12%	25%	2%

No allowance has been made, in calculations, for changes in asset allocations over time.

8.4 In the following three tabulations the sectors denoted G, H, J, L and N have been omitted because they started after 1985. The 46-year statistics for these sectors are dependent on the backdating described in Section 4 and might be misleading.

8.5 For sectors which started before 1985 (and for the section 8.3 "Balanced" and "Capital Stable" portfolios) the 46-year average arithmetic means have been:

Table 8.1 Arithmetic means over 46 years

Period	S	I	Q	P	F	C	B	D	W	X	Balncd	CapStb	Period
15	14.0%	10.8%	11.7%	9.6%	8.7%	8.8%	8.9%	8.6%	7.5%	5.5%	10.8%	9.4%	15
14	14.5%	10.8%	12.2%	9.9%	8.7%	8.8%	8.9%	8.7%	7.5%	5.5%	11.0%	9.5%	14
13	14.5%	10.3%	11.8%	10.1%	8.6%	8.9%	8.9%	8.7%	7.5%	5.5%	10.9%	9.4%	13
12	13.9%	9.5%	10.6%	9.9%	8.6%	8.9%	8.9%	8.7%	7.6%	5.6%	10.3%	9.1%	12
11	13.9%	9.0%	10.6%	9.7%	8.6%	8.8%	8.9%	8.7%	7.6%	5.6%	10.2%	9.0%	11
10	14.0%	8.7%	10.6%	9.8%	8.7%	8.8%	8.9%	8.7%	7.5%	5.6%	10.2%	9.1%	10
9	14.0%	8.5%	10.6%	9.9%	8.9%	8.8%	8.9%	8.7%	7.5%	5.6%	10.1%	9.1%	9
8	14.2%	8.9%	11.1%	10.0%	8.9%	8.8%	8.8%	8.7%	7.5%	5.6%	10.4%	9.2%	8
7	13.9%	9.3%	11.0%	10.1%	8.8%	8.7%	8.8%	8.6%	7.4%	5.5%	10.3%	9.2%	7
6	13.7%	9.5%	11.4%	10.2%	8.9%	8.6%	8.7%	8.6%	7.3%	5.5%	10.4%	9.2%	6
5	13.6%	9.9%	11.8%	10.3%	9.1%	8.5%	8.6%	8.5%	7.2%	5.5%	10.5%	9.3%	5
4	14.0%	10.0%	11.9%	10.4%	9.1%	8.4%	8.5%	8.4%	7.0%	5.4%	10.7%	9.4%	4
3	13.8%	10.1%	11.7%	10.5%	8.8%	8.3%	8.4%	8.4%	6.8%	5.3%	10.6%	9.2%	3
2	13.5%	10.3%	11.8%	10.4%	8.8%	8.2%	8.3%	8.3%	6.7%	5.2%	10.5%	9.1%	2
1	14.0%	11.1%	12.0%	10.2%	9.4%	8.1%	8.2%	8.2%	6.4%	5.0%	10.9%	9.5%	1
0	14.7%	11.5%	12.3%	10.1%	9.3%	7.8%	7.9%	8.0%	5.9%	4.6%	11.3%	9.6%	0

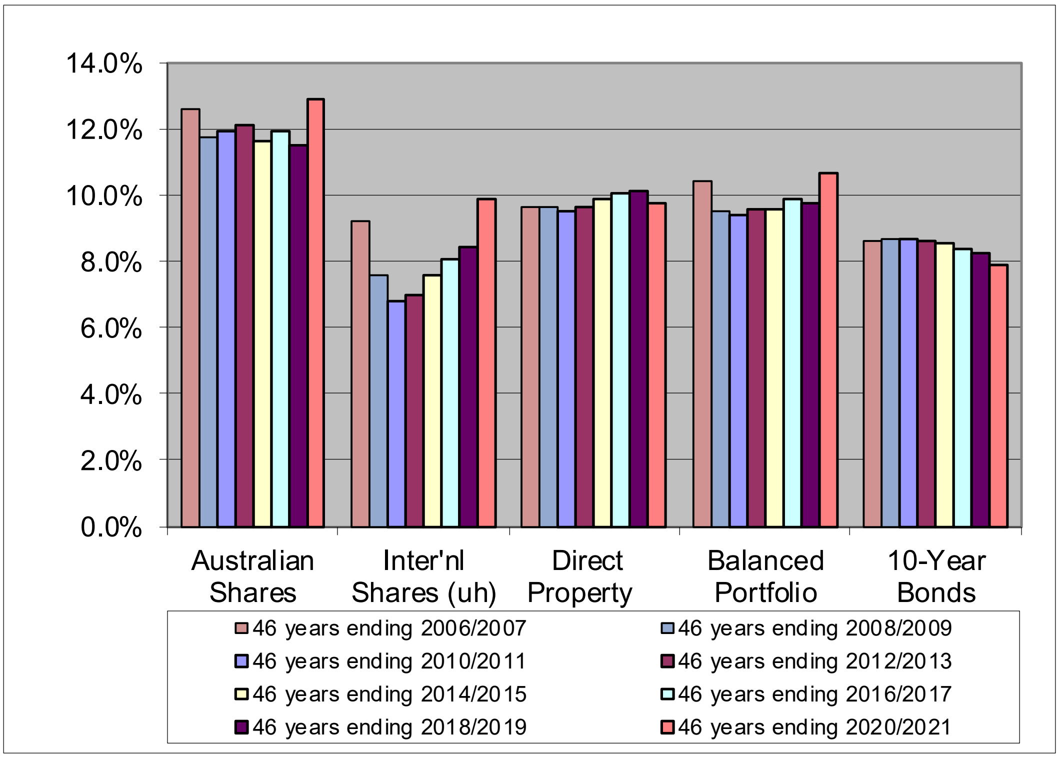
Note that for Periods 15 to 10 the average mean returns for Cash (C) and Bills (B) exceeded those for Bonds (D) and for Fixed Interest (F).

The 46-year average compound means have been:

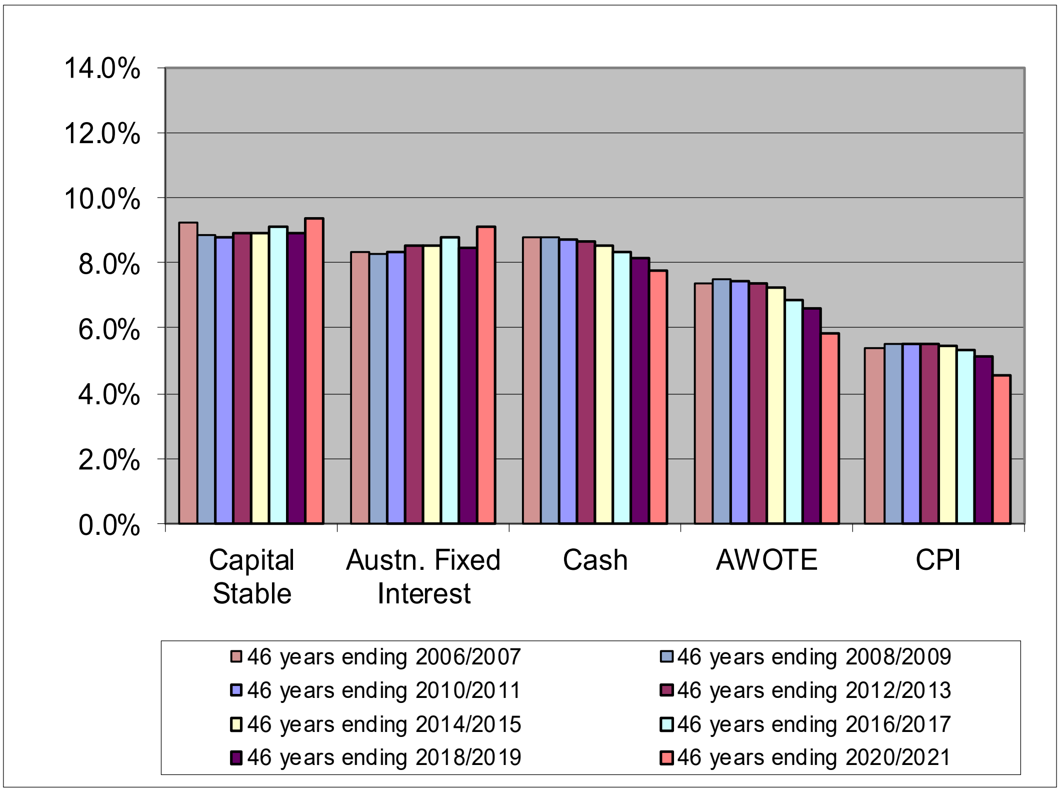
Table 8.2 Compound means over 46 years

Period	S	I	Q	P	F	C	B	D	W	X	Balncd	CapStb	Period
15	12.1%	9.2%	11.1%	9.4%	8.3%	8.7%	8.8%	8.6%	7.4%	5.4%	10.2%	9.1%	15
14	12.6%	9.2%	11.6%	9.7%	8.3%	8.8%	8.8%	8.6%	7.4%	5.4%	10.4%	9.2%	14
13	12.6%	8.7%	11.1%	9.8%	8.2%	8.8%	8.8%	8.6%	7.4%	5.4%	10.2%	9.1%	13
12	11.7%	7.6%	9.3%	9.6%	8.3%	8.8%	8.9%	8.6%	7.5%	5.5%	9.5%	8.8%	12
11	11.8%	7.1%	9.2%	9.4%	8.2%	8.8%	8.8%	8.7%	7.5%	5.5%	9.4%	8.8%	11
10	11.9%	6.8%	9.2%	9.5%	8.4%	8.7%	8.8%	8.7%	7.4%	5.5%	9.4%	8.8%	10
9	11.8%	6.6%	9.2%	9.6%	8.5%	8.7%	8.8%	8.7%	7.4%	5.5%	9.3%	8.8%	9
8	12.1%	7.0%	9.7%	9.6%	8.5%	8.7%	8.8%	8.6%	7.4%	5.5%	9.6%	8.9%	8
7	11.8%	7.3%	9.6%	9.7%	8.5%	8.6%	8.7%	8.6%	7.3%	5.5%	9.5%	8.9%	7
6	11.6%	7.6%	9.9%	9.8%	8.6%	8.5%	8.6%	8.6%	7.2%	5.5%	9.6%	8.9%	6
5	11.5%	8.0%	10.3%	10.0%	8.8%	8.5%	8.5%	8.5%	7.1%	5.4%	9.7%	9.1%	5
4	11.9%	8.1%	10.5%	10.0%	8.8%	8.3%	8.4%	8.4%	6.9%	5.4%	9.9%	9.1%	4
3	11.8%	8.2%	10.2%	10.1%	8.5%	8.2%	8.3%	8.3%	6.7%	5.2%	9.8%	8.9%	3
2	11.5%	8.4%	10.3%	10.1%	8.5%	8.1%	8.2%	8.2%	6.6%	5.1%	9.7%	8.9%	2
1	12.0%	9.3%	10.5%	9.9%	9.1%	8.0%	8.1%	8.1%	6.3%	4.9%	10.2%	9.3%	1
0	12.9%	9.8%	10.8%	9.7%	9.1%	7.7%	7.8%	7.9%	5.8%	4.6%	10.7%	9.4%	0

Figure 8.1 Compound means over 46 years



2006/2007 (for example) indicates average of years ending 30/9/06, 31/12/06, 31/3/07 & 30/6/07



2006/2007 (for example) indicates average of years ending 30/9/06, 31/12/06, 31/3/07 & 30/6/07

8.6 The 46-year average standard deviations have been:

Table 8.3 Standard deviations over 46 years

Period	S	I	Q	P	F	C	B	D	W	X	Balncd	CapStb	Period
15	12.1%	9.2%	11.1%	9.4%	8.3%	8.7%	8.8%	8.6%	7.4%	5.4%	10.2%	9.1%	15
14	12.6%	9.2%	11.6%	9.7%	8.3%	8.8%	8.8%	8.6%	7.4%	5.4%	10.4%	9.2%	14
13	12.6%	8.7%	11.1%	9.8%	8.2%	8.8%	8.8%	8.6%	7.4%	5.4%	10.2%	9.1%	13
12	11.7%	7.6%	9.3%	9.6%	8.3%	8.8%	8.9%	8.6%	7.5%	5.5%	9.5%	8.8%	12
11	11.8%	7.1%	9.2%	9.4%	8.2%	8.8%	8.8%	8.7%	7.5%	5.5%	9.4%	8.8%	11
10	11.9%	6.8%	9.2%	9.5%	8.4%	8.7%	8.8%	8.7%	7.4%	5.5%	9.4%	8.8%	10
9	11.8%	6.6%	9.2%	9.6%	8.5%	8.7%	8.8%	8.7%	7.4%	5.5%	9.3%	8.8%	9
8	12.1%	7.0%	9.7%	9.6%	8.5%	8.7%	8.8%	8.6%	7.4%	5.5%	9.6%	8.9%	8
7	11.8%	7.3%	9.6%	9.7%	8.5%	8.6%	8.7%	8.6%	7.3%	5.5%	9.5%	8.9%	7
6	11.6%	7.6%	9.9%	9.8%	8.6%	8.5%	8.6%	8.6%	7.2%	5.5%	9.6%	8.9%	6
5	11.5%	8.0%	10.3%	10.0%	8.8%	8.5%	8.5%	8.5%	7.1%	5.4%	9.7%	9.1%	5
4	11.9%	8.1%	10.5%	10.0%	8.8%	8.3%	8.4%	8.4%	6.9%	5.4%	9.9%	9.1%	4
3	11.8%	8.2%	10.2%	10.1%	8.5%	8.2%	8.3%	8.3%	6.7%	5.2%	9.8%	8.9%	3
2	11.5%	8.4%	10.3%	10.1%	8.5%	8.1%	8.2%	8.2%	6.6%	5.1%	9.7%	8.9%	2
1	12.0%	9.3%	10.5%	9.9%	9.1%	8.0%	8.1%	8.1%	6.3%	4.9%	10.2%	9.3%	1
0	12.9%	9.8%	10.8%	9.7%	9.1%	7.7%	7.8%	7.9%	5.8%	4.6%	10.7%	9.4%	0

8.7 For comparison with Table 8.2, the 28-year average compound means **for all sectors** have been:

Table 8.4 Compound means over 28 years

Period	S	I	Q	P	L	H	F	G	J
15	16.0%	12.0%	14.6%	10.4%	11.6%	13.4%	11.1%	11.3%	10.7%
14	15.7%	11.8%	14.6%	10.5%	11.5%	13.4%	10.8%	11.0%	10.5%
13	14.2%	11.0%	13.5%	10.5%	11.3%	13.0%	10.9%	10.9%	10.6%
12	11.3%	8.8%	10.2%	9.6%	11.0%	10.7%	11.1%	10.9%	10.5%
11	13.0%	9.1%	10.0%	8.8%	10.7%	11.9%	11.1%	10.6%	10.6%
10	13.1%	7.8%	9.5%	8.4%	10.2%	11.5%	10.5%	10.0%	10.1%
9	11.4%	7.1%	8.2%	8.3%	10.0%	10.8%	10.3%	9.6%	9.9%
8	11.7%	6.8%	8.7%	8.0%	9.8%	11.0%	10.0%	9.3%	9.7%
7	10.9%	6.3%	8.6%	7.8%	9.3%	10.3%	9.6%	8.8%	9.4%
6	9.7%	5.6%	8.3%	7.6%	8.9%	9.2%	9.3%	8.2%	9.1%
5	9.0%	5.7%	8.4%	7.2%	8.3%	8.7%	8.7%	7.7%	8.6%
4	9.3%	6.0%	8.8%	6.8%	7.9%	8.9%	8.5%	7.2%	8.4%
3	9.1%	5.8%	8.6%	6.6%	7.5%	8.7%	8.1%	6.7%	8.0%
2	9.6%	6.4%	8.8%	7.0%	7.1%	9.0%	7.6%	6.1%	7.4%
1	8.9%	6.6%	8.1%	7.7%	6.8%	8.9%	7.0%	5.6%	7.0%
0	9.2%	6.6%	7.9%	8.0%	6.5%	9.4%	6.4%	5.2%	6.6%

C	N	B	D	W	X	Balncd	CapStb	Period
10.0%	11.4%	10.0%	9.9%	6.3%	5.1%	13.1%	11.5%	15
9.9%	11.1%	9.8%	9.8%	6.2%	4.9%	12.9%	11.3%	14
9.7%	11.1%	9.7%	9.7%	6.0%	4.7%	12.2%	11.1%	13
9.4%	11.2%	9.5%	9.4%	5.7%	4.4%	10.3%	10.3%	12
9.0%	11.0%	9.0%	9.1%	5.4%	4.1%	10.9%	10.4%	11
8.5%	10.5%	8.6%	8.7%	5.1%	3.9%	10.5%	9.9%	10
8.2%	10.4%	8.3%	8.4%	4.9%	3.7%	9.6%	9.4%	9
7.9%	10.3%	8.0%	8.0%	4.8%	3.6%	9.6%	9.2%	8
7.4%	9.7%	7.4%	7.6%	4.7%	3.4%	9.0%	8.8%	7
6.9%	9.5%	6.9%	7.3%	4.6%	3.2%	8.3%	8.3%	6
6.5%	8.8%	6.5%	6.9%	4.4%	3.0%	7.9%	7.8%	5
6.1%	8.5%	6.1%	6.5%	4.2%	2.8%	8.0%	7.7%	4
5.5%	8.1%	5.5%	6.1%	4.1%	2.5%	7.7%	7.3%	3
5.1%	7.6%	5.1%	5.8%	3.9%	2.4%	7.9%	7.1%	2
4.8%	7.2%	4.8%	5.4%	3.9%	2.4%	7.6%	6.7%	1
4.6%	6.8%	4.5%	5.1%	4.0%	2.4%	7.6%	6.5%	0

8.8 For each of the 12 sectors in Table 8.2, the average 46-year returns are more stable than the average 28-year returns in Table 8.4. This is partly due to the longer averaging periods underlying Table 8.2 but, importantly, it is also due to Table 8.4 comprising relatively more of the down-slope of the economic cycles identified in Section 5. Because of the latter, the 9.3% to 11.1% F sector compound average returns for Periods 6 to 15 in Table 8.4 are inflated by capital appreciation and therefore exceed those in Table 8.2 that range from 8.2% to 8.6%.

8.9 For the reasons explained in the previous paragraph, the 46-year statistics form better indicators of long-term returns. Wherever possible it is desirable for the setting of long-term assumptions to analyse results over a full economic cycle (or over an integral number of economic cycles).

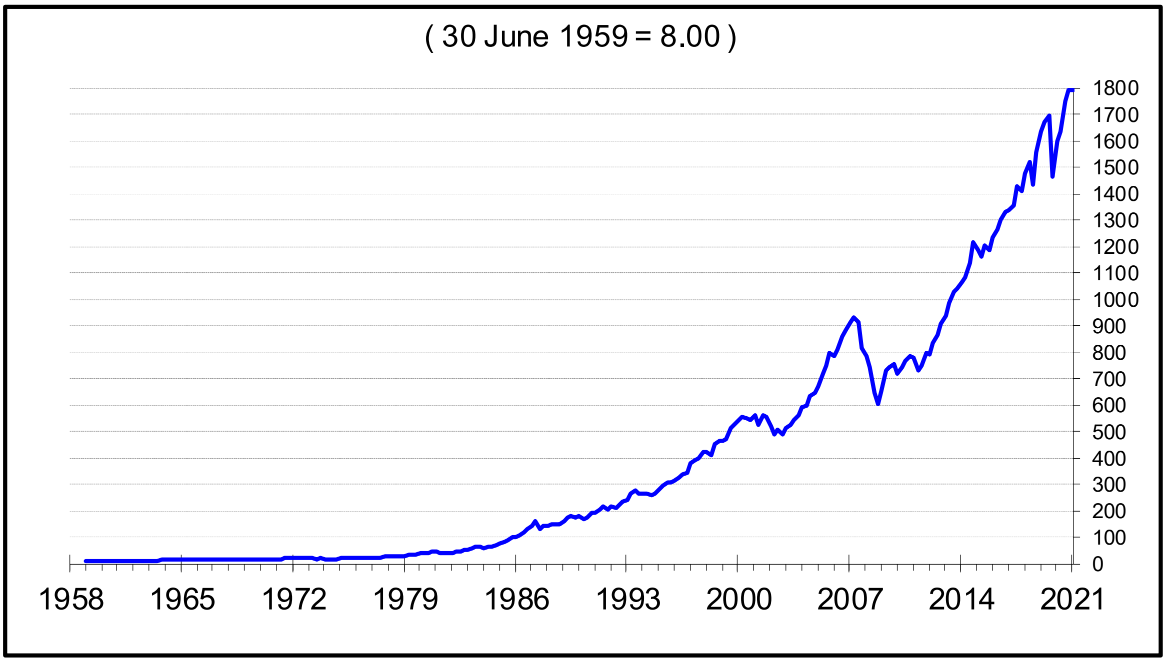
8.10 It should be noted that all individual results quoted in Sections 6 to 15, including those in this section, are the average of 4 annual results at quarterly intervals, **not individual results** for 46 or 28 years ending on one date.

9 Crashes and Recoveries

9.1 The “global financial crisis” (or “GFC”) was clearly a significant event during recent years. Was its impact right outside expectations, or was it predictable or consistent with past experience - never again or definitely again? How often can we expect the “perfect storm”?

9.2 Section 3.3 explained that the main database contains **annual forces at quarterly intervals**. Using this data and applying the fixed “Balanced” portfolio asset allocations explained in section 8.3, an array of quarterly unit prices was determined (starting from an initial value of 8.00 at 30 June 1959). Figure 9.1 below shows the results.

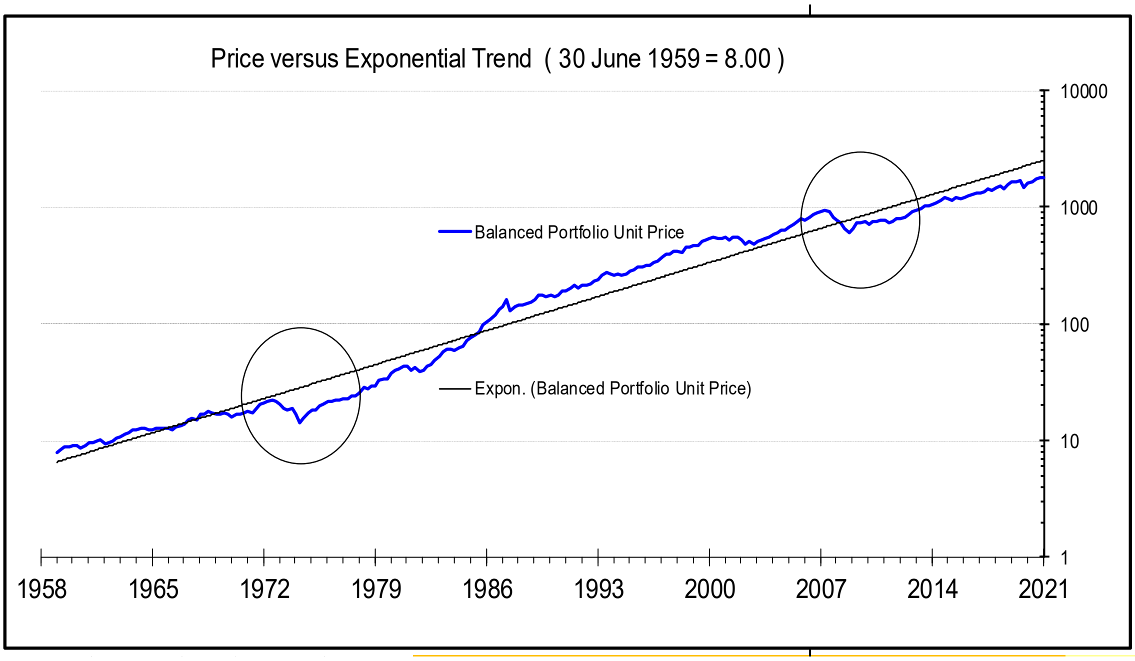
Figure 9.1 Balanced portfolio unit price



9.3 The Balanced portfolio unit price reached a quarterly maximum of 931.47 at 30 September 2007. In the following 18 months it fell 35.0% to a low of 605.70 at 31 March 2009. At 30 June 2017 the price had recovered 121.1% (from the low point of 8 years 3 months earlier) to 1339.28. By 31 March 2021 the price has risen to 1791.71, a rise of 195.8% in the 12 years since the 2009 low.

9.4 Figure 9.2 on the next page shows the same results as in Figure 9.1 but this time using a logarithmic scale. This more clearly demonstrates the frequency (or non-rareness) of the GFC and similar scale events throughout both the recent and the more distant past. This shows that, proportionately, the financial impact of the coronavirus global pandemic during 2020/2021 was far less severe than that of the global financial crisis – though it must be kept in mind that some of the impacts are hidden because the data is only available at quarterly intervals.

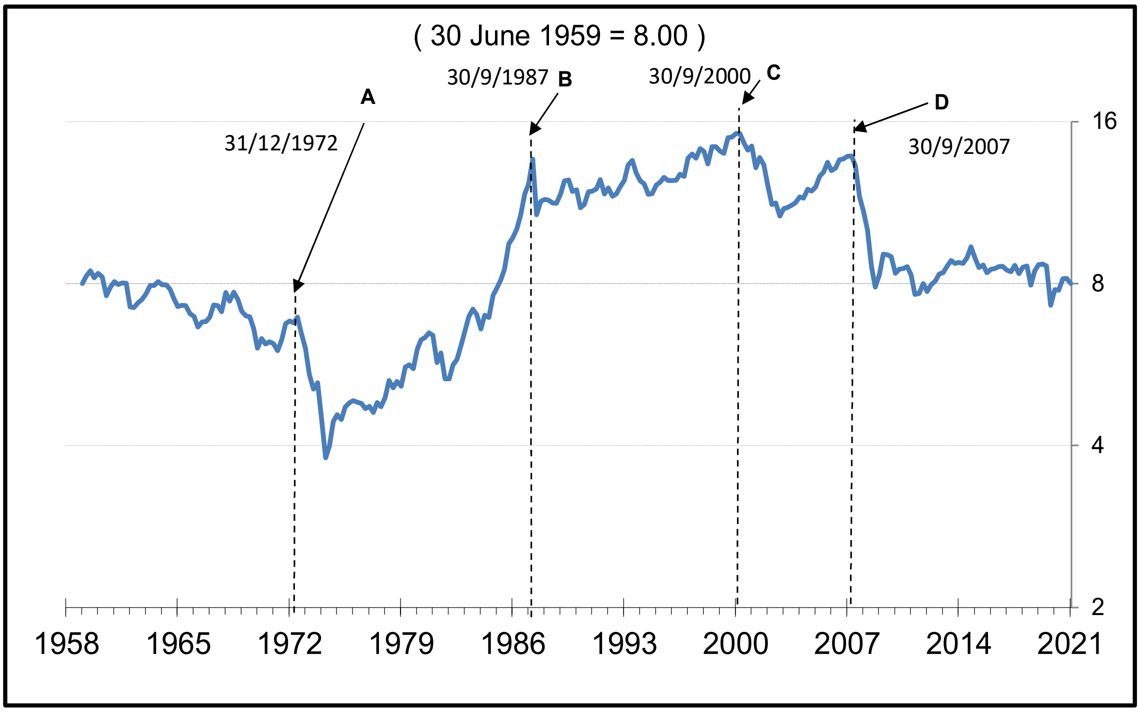
Figure 9.2 Balanced portfolio price on log scale



9.5 The exponential trend line in Figure 9.2, which presents as a straight line on this chart, indicates that the impact of the 2007 global financial crisis was not an isolated event. The two circles on the chart reveal very similar proportional falls (or 'crashes').

9.6 In the **sixty-two years** between 30/6/59 and 30/6/21 the Balanced portfolio return averaged **9.12% pa compound**. To help identify the peaks in the data, the Balanced unit prices were discounted by this long-term rate. These "discounted unit prices" can be interpreted as a series of present values as at 30/6/59. Figure 9.3 below shows the results.

Figure 9.3 Balanced portfolio "discounted price"

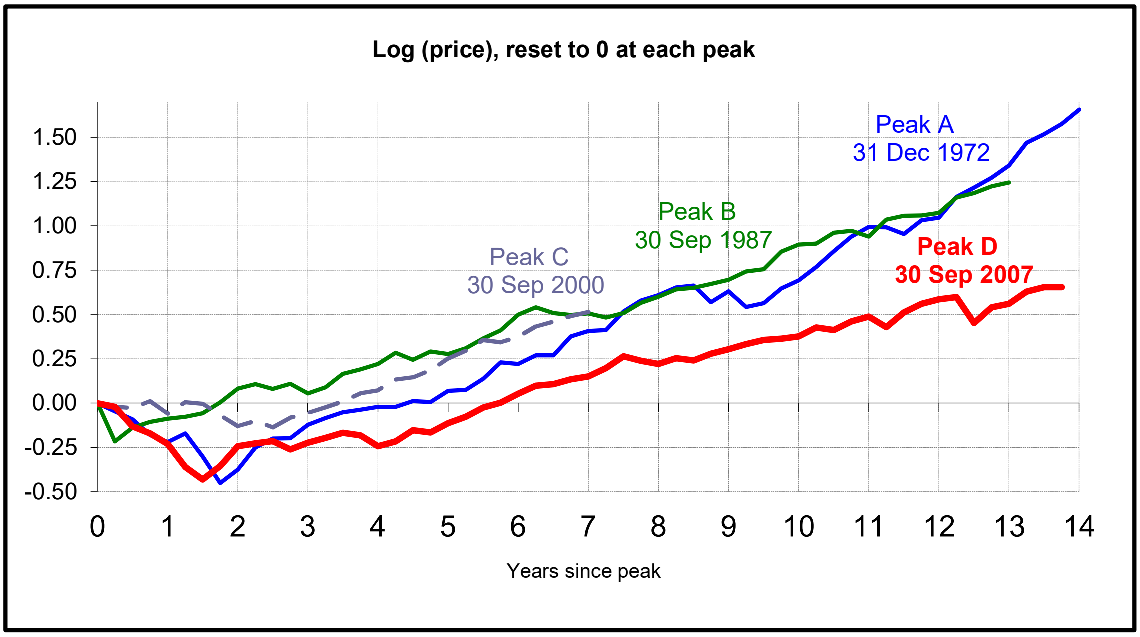


9.7 The four peaks in the discounted price (identified as A, B, C and D in Figure 9.3) occurred at 31/12/72, 30/9/87, 30/9/00 and 30/9/07 respectively. The following analyses what happened in the period immediately after these peaks:

Peak	Period	Months	Fall in Price
A	31/12/72 – 30/9/74	21	36.2%
B	30/9/87 – 31/12/87	3	19.4%
C	30/9/00 – 31/3/03	30	12.7%
D	30/9/07 – 31/3/09	18	35%

In Figure 9.4 below, the log of the unit price (not the log of the discounted unit price) has been reset to 0 at each of the peaks A, B, C and D. A close examination of Figure 9.4 shows that the "peak C" curve based on the log of the unit prices was almost flat during the first seven quarters after the peak, though the "discounted price" decreased by 12.5% during these seven quarters. The "peak C" curve ceases after 7 years when it reaches peak D. The "peak B" curve ceases after 13 years when it reaches peak C and the peak D curve ceases after 13.75 years at 30 June 2021.

Figure 9.4 Balanced portfolio with price reset

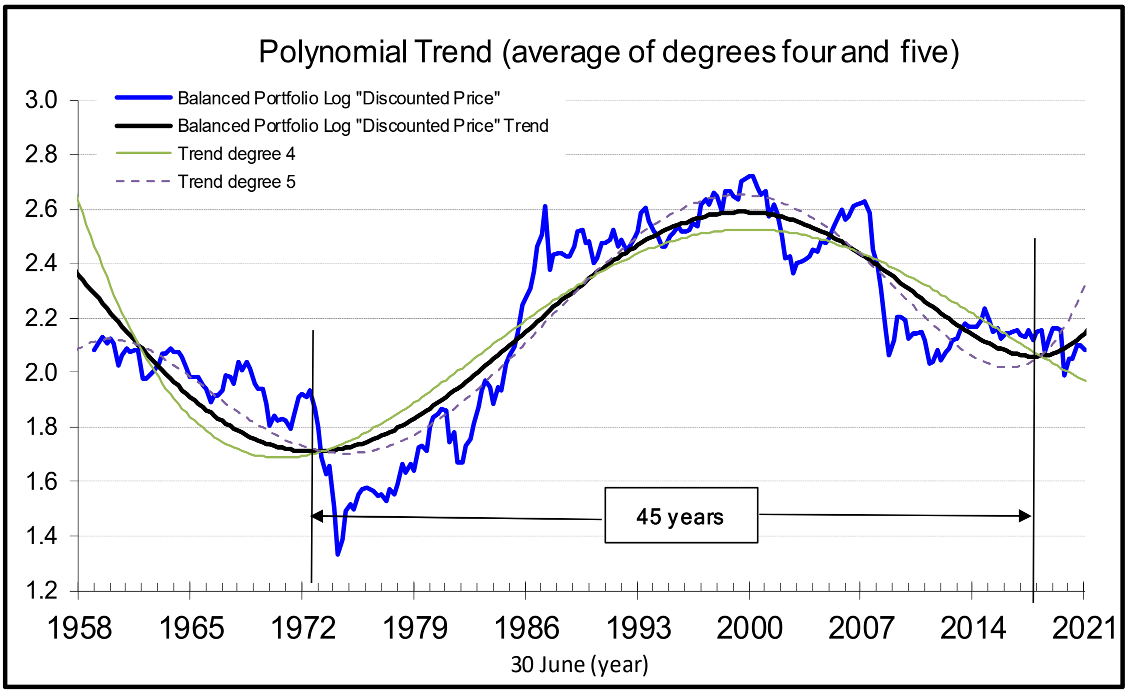


9.8 The chart above indicates that the impact of the global financial crisis on a Balanced portfolio was far more severe than the falls that occurred after the 30/9/87 and 30/9/00 peaks. However, its impact and initial recovery were very similar to the 1974 market crash. The big economic difference between 1974 and 2009, which is not evident from the above charts, is that annual rates of salary increase (as measured by AWOTE changes) during the two years before and after 31/12/72 were 11.8% and 8.2%, and 15.3% and 28.3% respectively. In the two years before and after 30/9/07 the annual rates of salary increase (as measured by AWOTE changes) were only 2.8% and 5.4%, and 4.7% and 5.2%.

9.9 The other difference between 1974 and 2009, which is evident from the blue and red lines on the above chart, is that returns during the latter period of recovery (to 30 June 2021) have lagged significantly behind the post 1974 period (primarily due to low returns on Australian Fixed Interest investments and Cash and, post 31 December 2019, due to the economic consequences of the coronavirus global pandemic).

9.10 As a consequence of its design, the "discounted unit price" described in section 9.6, starts and ends at 8.00. In the following Figure 9.5 a polynomial trend line of the log of the discounted price is shown. It is equivalent to the average of the trends for degrees four and five. This particular trend line was chosen because the degree four trend did not fit the data well at the start and the degree five trend did not fit the data well at the end.

Figure 9.5 Balanced portfolio log "discounted price" and trend



9.11 Figure 9.5 shows that the period between minima in the trend line was 45.0 years (from 31 March 1973 to 31 March 2018). This adds support to the choice of 46 years in section 5.19 for the "average" past economic cycle. It should be noted that the methodology used to determine this cycle length is very different to that used in section 5.19. The latter is based on forces and sine curves, whereas this is based on logs of discounted prices and polynomial trends. Nevertheless, the conclusion is very similar.

9.12 Applying the same analysis as in section 9.11, but using the "Capital Stable" portfolio asset allocations explained in section 8.3, resulted in the period between minima in the trend line increasing to 46.5 years (dating from 31 March 1973 to 30 September 2019).

10 Outliers

10.1 Outliers can introduce bias into the sample means and standard deviations and in particular can have a significant effect on skewness and kurtosis.

10.2 Grenfell (2007) considered the F sector (Australian fixed interest securities) and explained that the annual force for the year ending 30 September 1973 represented 3.39 standard deviations below the mean. If the F sector had a normal distribution this value had about one chance in 2,900 of occurring. It was compared with the most extreme outliers for Australian shares, in September 1974, of 2.69 standard deviations below the mean and, in September 1987, of 2.53 standard deviations above the mean.

10.3 However, during 2008/09 a far more dramatic outlier occurred.

10.4 Consider the following 46-year annual forces for years ending 30 September, 31 December, 31 March and 30 June for the Q sector (listed Australian Property Trusts). The period chosen corresponds to "Period 0" in section 2.5. The blue shading indicates backdated data.

Table 10.1 Property trust (Q sector) annual forces

Year Ending	30-Sep	31-Dec	31-Mar	30-Jun	Year Ending	30-Sep	31-Dec	31-Mar	30-Jun
1975/76	18.1%	16.9%	12.0%	12.6%	1998/99	13.6%	16.5%	4.5%	4.2%
1976/77	13.3%	6.2%	6.7%	6.6%	1999/00	0.0%	-5.1%	1.0%	11.3%
1977/78	4.6%	15.8%	25.2%	29.5%	2000/01	8.7%	16.4%	12.0%	13.0%
1978/79	31.2%	26.4%	19.3%	14.4%	2001/02	15.1%	13.8%	16.0%	14.2%
1979/80	13.6%	10.0%	15.2%	10.6%	2002/03	11.3%	11.2%	13.0%	11.5%
1980/81	4.7%	6.2%	25.1%	22.0%	2003/04	6.1%	8.4%	13.1%	15.9%
1981/82	17.9%	27.8%	-3.0%	3.5%	2004/05	25.5%	27.9%	18.2%	16.9%
1982/83	13.8%	5.0%	18.1%	21.3%	2005/06	15.5%	12.0%	17.0%	16.6%
1983/84	29.9%	40.7%	34.6%	30.2%	2006/07	22.8%	29.3%	25.2%	23.4%
1984/85	22.0%	9.6%	12.5%	11.1%	2007/08	18.3%	-8.7%	-27.7%	-47.4%
1985/86	5.4%	5.1%	14.3%	21.3%	2008/09	-54.1%	-80.5%	-86.9%	-54.7%
1986/87	24.5%	30.3%	26.4%	34.6%	2009/10	-26.2%	9.1%	35.1%	18.5%
1987/88	42.7%	5.6%	8.8%	-2.8%	2010/11	-4.6%	-0.7%	4.6%	5.7%
1988/89	-20.6%	14.9%	1.2%	-1.1%	2011/12	-6.5%	-1.6%	1.7%	10.4%
1989/90	10.0%	2.3%	9.6%	14.2%	2012/13	25.4%	28.4%	26.6%	21.5%
1990/91	8.8%	8.3%	12.1%	7.4%	2013/14	15.2%	7.0%	4.8%	10.5%
1991/92	4.6%	18.3%	10.0%	13.8%	2014/15	11.6%	23.7%	29.6%	18.4%
1992/93	15.1%	6.8%	15.5%	15.8%	2015/16	18.3%	13.4%	10.8%	22.0%
1993/94	23.7%	26.3%	16.9%	9.4%	2016/17	19.0%	12.4%	6.1%	-5.8%
1994/95	-1.3%	-5.7%	2.5%	7.6%	2017/18	-2.0%	6.2%	-0.1%	12.4%
1995/96	9.4%	12.0%	3.7%	3.6%	2018/19	12.4%	3.2%	23.0%	17.7%
1996/97	11.6%	13.5%	18.1%	25.1%	2019/20	16.9%	17.9%	-37.6%	-23.2%
1997/98	21.7%	18.5%	23.6%	9.5%	2020/21	-17.2%	-4.0%	37.4%	19.0%

30-Sep	31-Dec	31-Mar	30-Jun		
Statistics for last 46 years:					Average across
10.21%	10.38%	10.35%	10.26%	mu	10.30%
16.25%	17.37%	20.14%	16.52%	sigma	17.57%
-165%	-311%	-287%	-240%	skewness	-251%
489%	1639%	1199%	736%	kurtosis	1016%

Modified statistics for last 46 years:					Average across
10.21%	10.38%	10.35%	10.26%	mu	10.30%
16.25%	17.37%	19.79%	16.52%	sigma	17.48%
-165%	-311%	-264%	-240%	skewness	-245%
489%	1639%	1006%	736%	kurtosis	968%

10.5 The annual force for the year ending 31 March 2009 was –86.9% (boxed). This represents 4.8 standard deviations below the 46-year 31 March mean. If the Q sector annual forces had a normal distribution (which they clearly haven't) this value has about one chance in 1,442,000 of occurring. The Q sector annual forces for the years ending 30 September 2008, 31 December 2008, 31 March 2009 and 30 June 2009 all represent more than 3.9 standard deviations below their means.

10.6 The Q sector skewness and kurtosis for the 46 years ending 2020/2021 averaged **-251% and 1016%** respectively. However, the outlier of -86.9% (for the year ending 31 March 2009) significantly affects these results.

10.7 To indicate the sensitivity of the skewness and kurtosis to the March 2009 outlier, if this force had been 6% greater (i.e. –80.9%) and the force for the previous year had been 6% less (i.e. -33.7%), then the modified average skewness and kurtosis for the 44 years ending 2008/2009 would have been **-245% and 968%** respectively. This is equivalent to changing just one quarterly unit price (in this case, decreasing the 31 March 2008 unit price). The assumptions set out in Section 16 are not based on the modified statistics. Note that changes of 6% greater and 6% less are the largest integral percentage changes possible without changing the quarterly ranks of the tabulated forces.

10.8 By any reasonable yardstick, the experience of the Q sector twelve years ago was extreme. It is probable this experience was related to the high levels of debt exposure apparent in Australian listed property investments. When setting assumptions for the future the extreme results need to be carefully recognised. The specific adjustments now made when setting the Q sector risk margin and coefficient of variation have already been explained in Sections 6 and 7. The specific adjustments considered for the Q sector skewness and kurtosis are explained in Sections 11 and 12. Section 13.4 explains why rank cross-correlations are not distorted by outliers and hence do not require specific adjustments.

10.9 Chan, Ng and Tong (2006) make a number of useful observations about outliers and explain that, whether or not it is appropriate to adjust the data for the outliers, depends on the purpose to which the model so derived will be used.

REMEMBER

> *"A* ***'black swan'*** *event is a seemingly inconceivable event that occurs infrequently and has massive impact. Quantitative risk modelling, with its bias in historic information would not be adequate to cover the risk of these future events that have not been conceived, especially when using nice smooth mathematical functions. The tails and extremes of events most likely don't follow nice smooth laws; across the range of possible outcomes, it is extremely unlikely to get a 'one size fits all' distribution. Nice smooth laws tend to assume rational behaviour underlying them. The world can be discontinuous, which most mathematical models are not, and this is where unpredictability rules and chaos reigns. …*
>
> *Many actuarial methods involve backward looking, using past data which is out of date, sparse and perhaps no longer relevant. Future trends may not be predictable from the past (i.e. discontinuities / hysteresis)."*
>
> *Waite (2009)*

11 Skewness

11.1 Skewness characterises the degree of asymmetry of a distribution around its mean. Positive skewness indicates a distribution with an asymmetric tail extending towards more positive values. Negative skewness indicates a distribution with an asymmetric tail extending towards more negative values.

11.2 As explained in section 2.11, the skewness results shown in this section, and the kurtosis, cross-correlation and auto-correlation results shown in Sections 12 to 15 are based on annual forces.

11.3 When using stochastic models, it is often desired to convert assumptions and/or model results from rates to forces and vice versa, and to combine the results from a number of sectors to form or approximate results for a portfolio or composite sector. Such changes are often based on the assumption, or an implied assumption, of a normal distribution for the forces and a lognormal distribution for the rates. However, if non-zero values for skewness and kurtosis are used as input to a stochastic model then formulae (based on a normal distribution for the forces and a lognormal distribution for the rates) will not hold.

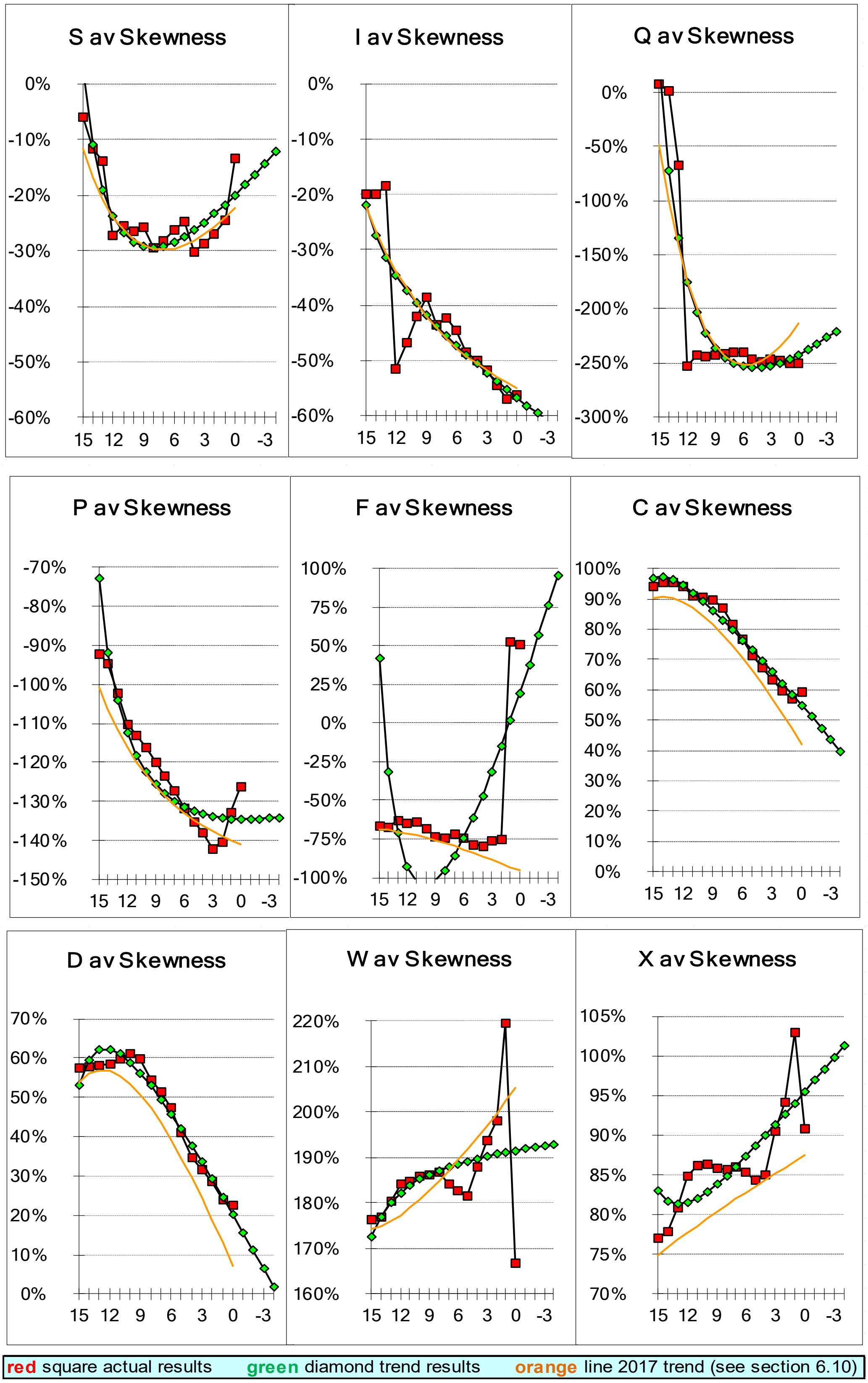
11.4 For example, when using dependent variables, and with non-zero values for skewness and kurtosis, it is not easy to determine:

- Long-term estimates of standard deviations of **portfolio or composite sector** returns from long-term estimates of standard deviations of returns for **each individual sector**, or
- Long-term estimates of means and/or standard deviations of **forces** for each individual sector from long-term estimates of means and/or standard deviations of **rates** for each individual sector.

11.5 However, under suitable assumptions, formulae can be determined to resolve these issues. For example, Appendix A of Grenfell (2005) contains formulae for statistical conversion between rates and forces of return for use when skewness and/or kurtosis are not zero.

11.6 On the following pages, Figure 11.1 and Table 11.1 illustrate the skewness results for each sector.

Figure 11.1 Skewness over 46 years



X-axis: Period 15 = average 46 years ending 30/9/05, 31/12/05, 31/3/06 and 30/6/06
Period 0 = average 46 years ending 30/9/20, 31/12/20, 31/3/21 and 30/6/21 (see section 6.8)

Table 11.1 Skewness over 46 years

Actual and *Quadratic Trend*

Period	S	I	Q	P	L	H	F	G
15	-6%	-20%	7%	-92%	81%	-53%	-66%	41%
14	-12%	-20%	1%	-95%	82%	-56%	-67%	41%
13	-14%	-19%	-67%	-102%	82%	-53%	-63%	42%
12	-27%	-51%	-252%	-110%	79%	-80%	-65%	38%
11	-25%	-47%	-243%	-113%	78%	-75%	-64%	37%
10	-26%	-42%	-244%	-116%	79%	-74%	-68%	38%
9	-26%	-38%	-243%	-120%	78%	-71%	-73%	37%
8	-29%	-44%	-242%	-123%	76%	-77%	-74%	37%
7	-28%	-42%	-241%	-127%	75%	-78%	-72%	37%
6	-26%	-45%	-240%	-132%	75%	-79%	-74%	38%
5	-25%	-49%	-247%	-135%	73%	-80%	-78%	37%
4	-30%	-50%	-249%	-138%	70%	-84%	-79%	36%
3	-29%	-52%	-247%	-142%	69%	-83%	-76%	40%
2	-27%	-54%	-248%	-140%	70%	-81%	-75%	43%
1	-25%	-57%	-251%	-133%	79%	-82%	53%	63%
0	-14%	-56%	-251%	-126%	80%	-73%	51%	64%
0	*-20%*	*-57%*	*-243%*	*-134%*	*76%*	*-79%*	*20%*	*59%*
-1	*-18%*	*-58%*	*-238%*	*-134%*	*77%*	*-78%*	*38%*	*63%*
-2	*-16%*	*-60%*	*-233%*	*-134%*	*78%*	*-77%*	*57%*	*67%*
-3	*-14%*	*-61%*	*-227%*	*-134%*	*78%*	*-76%*	*76%*	*71%*
-4	*-12%*	*-62%*	*-221%*	*-134%*	*79%*	*-76%*	*96%*	*76%*

J	C	N	B	D	W	X	Period
-58%	95%	-20%	108%	57%	176%	77%	15
-60%	95%	-20%	108%	58%	177%	78%	14
-60%	96%	-20%	108%	58%	180%	81%	13
-56%	94%	-22%	108%	58%	184%	85%	12
-59%	91%	-21%	105%	60%	185%	86%	11
-64%	91%	-25%	104%	61%	186%	86%	10
-70%	90%	-32%	104%	60%	186%	86%	9
-72%	87%	-35%	100%	54%	187%	86%	8
-72%	82%	-34%	95%	51%	184%	86%	7
-75%	77%	-38%	90%	48%	183%	85%	6
-80%	72%	-38%	85%	41%	182%	84%	5
-82%	67%	-38%	81%	35%	188%	85%	4
-76%	64%	-31%	77%	32%	194%	90%	3
-71%	60%	-28%	74%	28%	198%	94%	2
34%	57%	-1%	71%	24%	220%	103%	1
29%	59%	2%	70%	23%	167%	91%	0
5%	*55%*	*-7%*	*67%*	*20%*	*192%*	*95%*	*0*
20%	*51%*	*-1%*	*64%*	*16%*	*192%*	*97%*	*-1*
36%	*48%*	*5%*	*60%*	*11%*	*192%*	*98%*	*-2*
52%	*44%*	*10%*	*56%*	*6%*	*193%*	*100%*	*-3*
68%	*40%*	*16%*	*52%*	*2%*	*193%*	*101%*	*-4*

11.7 For all sectors, the quadratic trend Period –2 results are the skewness assumptions that are tabulated in Section 16. For the F and J sectors (Australian and International Fixed Interest) the Period 2 result (shaded green) suddenly moves from negative to positive for Period 1. This is due to the "old data dropping off" explained in section 2.9. However, the Period -2 trend results seem consistent with the Periods 1 and 0 new data.

REMEMBER

*"**'Spherical cow'** is metaphor for highly simplified models of reality. It is from a mathematical joke where the punch line is the mathematician's solution to improve milk production and starts with "Consider a spherical cow … ". The point of the joke is that model builders will often reduce a problem to its simplest form in order to make calculations feasible, even though such simplification may hinder the model's application to reality."*

Waite (2009)

12 Kurtosis

12.1 Kurtosis characterizes the relative peakedness or flatness of a distribution compared to the normal distribution. Positive kurtosis indicates a relatively peaked distribution. Negative kurtosis indicates a relatively flat distribution.

12.2 The tails of a distribution are significantly influenced by the kurtosis and to a lesser extent the skewness.

12.3 Figure 12.1 and Table 12.2 illustrate the kurtosis results for each sector.

12.4 For illustrative purposes it is informative to classify and rank the skewness and kurtosis assumptions (tabulated in Section 16) as follows:

(a) **minimal** between -75% and +75% skewness and kurtosis.
(b) **moderate** between -76% and -199% skewness or kurtosis, or between +76% and +199% skewness or kurtosis.
(c) **high** between 200% and 600% kurtosis.
(d) **extreme** over 600% kurtosis.

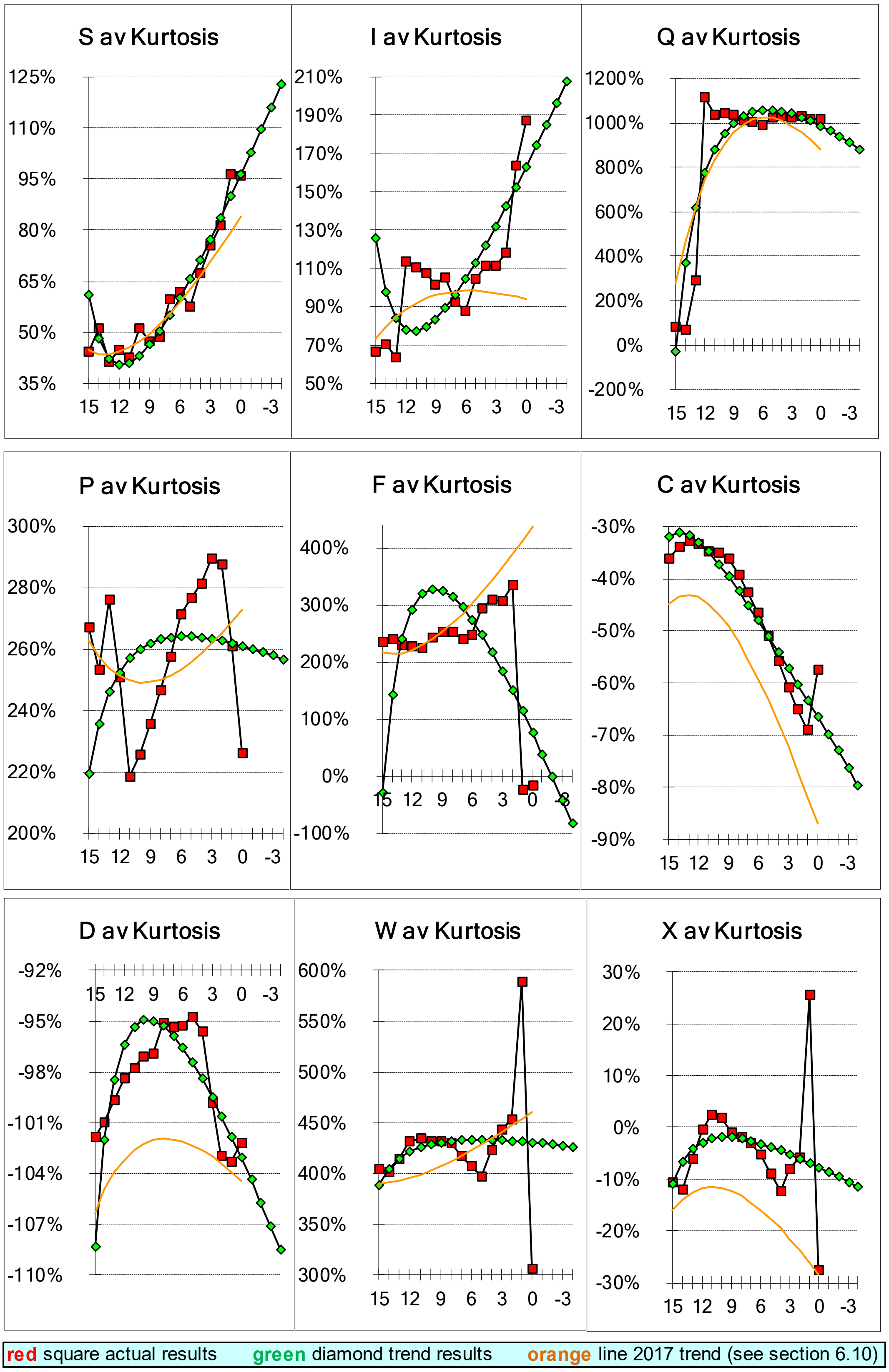
The boundaries for the last two classifications recognise that an exponential distribution has a skewness of exactly 2 and a kurtosis of exactly 6 and that the normal distribution fails to give correct probabilities for the tail of this distribution.

Table 12.1 Skewness and kurtosis classification

Sector		Skewness	Kurtosis	Classification
N	Inflation Linked Bond	5%	5%	Minimal
J	Inter'nl Fixed Interest	36%	18%	Minimal
C	Cash	48%	-73%	Minimal
F	Austn. Fixed Interest	57%	-2%	Minimal
B	90-Day Bills	60%	-46%	Minimal
H	Hedged Intl Shrs	-77%	185%	Moderate
I	Inter'nl Shares (uh)	-60%	185%	Moderate
S	Australian Shares	-16%	109%	Moderate
CapStb	Capital Stable	5%	135%	Moderate
D	10-Year Bonds	11%	-106%	Moderate
G	Semi govt (0 to 3 yrs	67%	-81%	Moderate
L	Loans/corp credit	78%	-16%	Moderate
X	CPI	98%	-10%	Moderate
P	Direct Property	-134%	259%	High
Balncd	Balanced	-77%	238%	High
W	AWOTE	192%	429%	High
Q	Property Trusts	-233%	963%	Extreme

Although this classification is only illustrative, the number of assumptions that fall outside the minimal classification sounds a warning for those who use normal or lognormal models in relation to investment performance. In the last four years the classification of a number of sectors has changed – the most significant being F and J sectors moving from "High" to "Minimal".

Figure 12.1 Kurtosis over 46 years



X-axis: Period 15 = average 46 years ending 30/9/05, 31/12/05, 31/3/06 and 30/6/06
Period 0 = average 46 years ending 30/9/20, 31/12/20, 31/3/21 and 30/6/21 (see section 6.8)

Table 12.2 Kurtosis over 46 years

Actual and *Quadratic Trend*

Period	S	I	Q	P	L	H	F	G
15	44%	66%	85%	267%	-9%	71%	235%	-22%
14	51%	71%	72%	253%	-5%	77%	241%	-21%
13	42%	64%	288%	276%	-5%	75%	229%	-22%
12	45%	113%	1112%	251%	-8%	129%	228%	-23%
11	43%	111%	1038%	218%	-8%	123%	223%	-27%
10	51%	107%	1043%	226%	-3%	122%	242%	-25%
9	47%	101%	1037%	236%	1%	114%	252%	-22%
8	49%	106%	1011%	246%	2%	125%	252%	-25%
7	59%	93%	1006%	258%	-1%	124%	240%	-31%
6	62%	88%	990%	271%	0%	125%	248%	-37%
5	57%	104%	1021%	277%	0%	143%	295%	-37%
4	68%	111%	1033%	281%	-5%	155%	310%	-42%
3	76%	111%	1021%	290%	-11%	155%	308%	-47%
2	81%	118%	1027%	288%	-14%	148%	334%	-51%
1	97%	164%	1018%	261%	-11%	177%	-23%	-70%
0	96%	187%	1016%	226%	-10%	174%	-17%	-71%
0	*96%*	*163%*	*986%*	*261%*	*-12%*	*173%*	*77%*	*-69%*
-1	*103%*	*174%*	*963%*	*260%*	*-14%*	*179%*	*38%*	*-75%*
-2	*109%*	*185%*	*938%*	*259%*	*-16%*	*185%*	*-2%*	*-81%*
-3	*116%*	*196%*	*910%*	*258%*	*-18%*	*192%*	*-42%*	*-87%*
-4	*123%*	*208%*	*882%*	*257%*	*-20%*	*198%*	*-83%*	*-94%*

J	C	N	B	D	W	X	Period
196%	-36%	43%	13%	-102%	404%	-10%	15
207%	-34%	44%	15%	-101%	402%	-12%	14
207%	-33%	41%	17%	-100%	415%	-6%	13
182%	-34%	43%	18%	-98%	431%	0%	12
186%	-35%	28%	16%	-98%	434%	2%	11
207%	-35%	42%	15%	-97%	431%	2%	10
219%	-36%	41%	13%	-97%	432%	-1%	9
224%	-39%	42%	8%	-95%	430%	-2%	8
218%	-43%	22%	2%	-95%	418%	-3%	7
229%	-47%	27%	-4%	-95%	407%	-5%	6
278%	-51%	38%	-10%	-95%	398%	-9%	5
291%	-56%	40%	-17%	-96%	422%	-12%	4
273%	-61%	32%	-24%	-100%	443%	-8%	3
278%	-65%	34%	-29%	-103%	453%	-6%	2
1%	-69%	8%	-33%	-103%	589%	25%	1
8%	-58%	5%	-28%	-102%	306%	-28%	0
83%	*-67%*	*13%*	*-35%*	*-103%*	*431%*	*-8%*	*0*
51%	*-70%*	*9%*	*-41%*	*-104%*	*430%*	*-9%*	*-1*
18%	*-73%*	*5%*	*-46%*	*-106%*	*429%*	*-10%*	*-2*
-16%	*-76%*	*1%*	*-51%*	*-107%*	*428%*	*-10%*	*-3*
-51%	*-80%*	*-3%*	*-57%*	*-108%*	*427%*	*-11%*	*-4*

12.5 Except for the Q sector, the quadratic trend Period –2 results are the kurtosis assumptions that are tabulated in Section 16. For the Q sector (listed Australian Property Trusts) the standard error of the trend-fitting was nearly twice that for all other sectors – hence the Period -1 trend result was used because that was more consistent with all the actual results since period 12.

REMEMBER

"Small changes in average behaviour do not give good evidence of what's happening in the tails, e.g. a smaller number of large banks may increase stability on average, but due to globalisation and interconnectedness, the tail risk becomes extreme (e.g. Fannie May and Freddie Mac). Even a fat-tailed distribution may understate the frequency and size of some risks."

Waite (2009)

13 Cross-correlations

13.1 The methodology used for risk margins, CoVs, skewness and kurtosis is detailed in Section 2. However, for correlations it is essential that projected statistics are entirely consistent with each other over sectors (for cross-correlations) or over time (for auto-correlations). To obtain this greater consistency, the methodology was therefore slightly varied for correlations.

13.2 After step 2 in Section 2 the database was extended by adding on 6 years of "projected" annual forces for each quarter date. For this purpose, the "projected" values were obtained from the projected trend risk margins (see Table 6.4) by adding back the fixed Bond base value (5.0% per annum) and converting to forces.

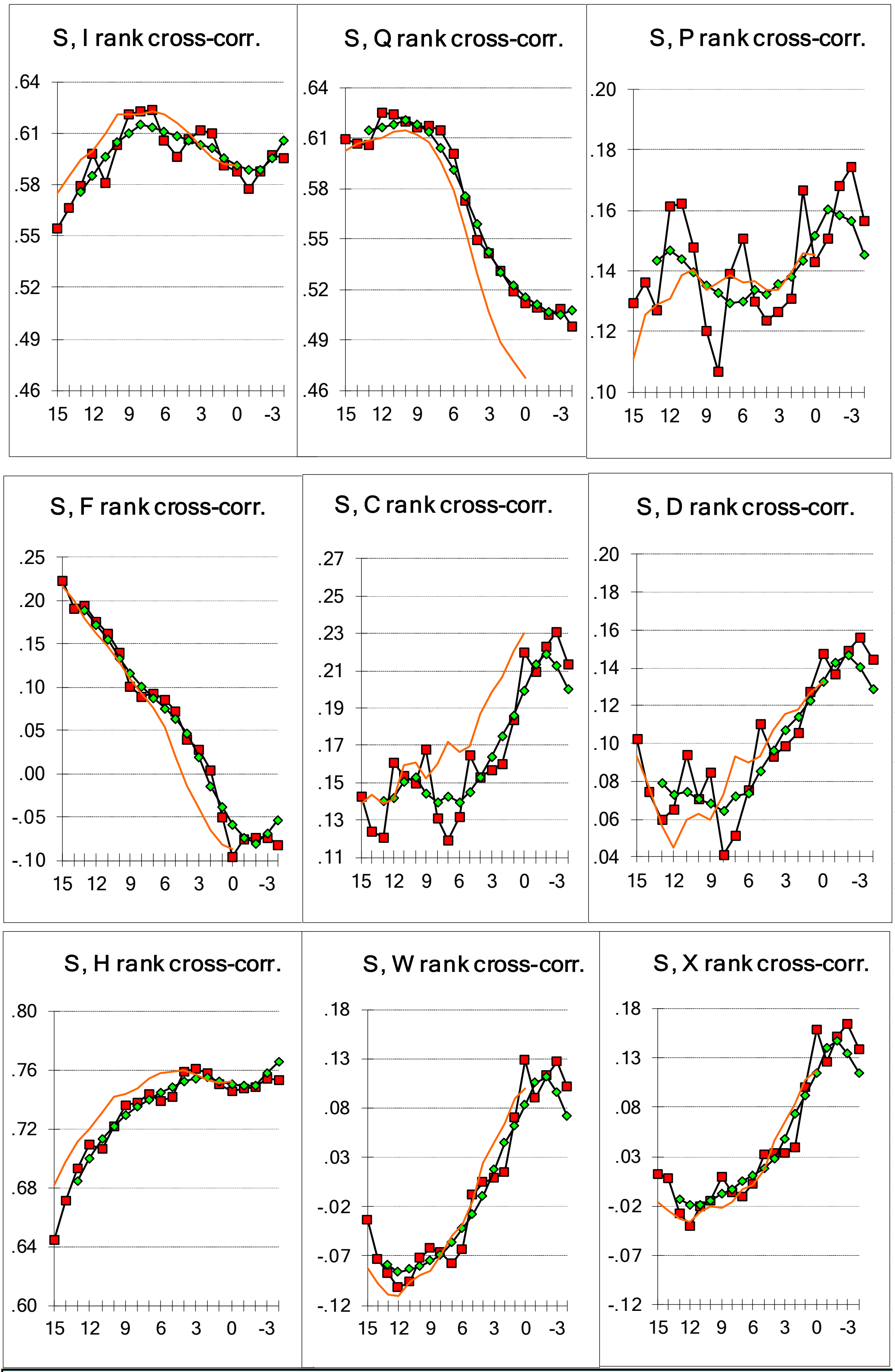
13.3 After the adjustments in 13.2 above, the extended database of each sector covered 67 years for each of the four quarter-end dates. (For the June quarter-end-dates the extended database covered 68 years but the first year was disregarded). Steps 3 and 4 in Section 2 were then applied and step 5 was replaced by central 5-year running averages.

13.4 When values are changing rapidly from one period to the next the use of 5-year running averages may slightly distort results. An alternative might be to use a regression trend value at "Period -2". However, having quadratic (or higher degree) trends at both steps 2 and 5 could result in hidden distortions in end results. Such an alternative was attempted in Grenfell (2009) for auto-correlations and found to produce results, up to a lag of 20 years, less than 3% (and usually less than 1%) different from using the more straightforward central 5-year average.

13.5 It is well documented that sample correlations can be distorted by outliers and non-zero skewness and kurtosis. To overcome this problem, for cross-correlations, 67-year ranks of the extended database forces were used as well as the actual forces. Thus, two sets of cross-correlations were obtained, one based on ranks and one based on actual (i.e. actual plus backdated plus projected) forces. Correlations based on the former will be referred to as "rank" correlations, and correlations based on the latter will be referred to as "standard" correlations. In the *Austmod* investment simulation model, the rank cross-correlations are used because they are not distorted by outliers or by non-zero skewness or kurtosis (because the ranks of the forces are uniformly distributed) and the impacts of skewness and kurtosis are separately allowed for in the model.

13.6 On the following pages, Figure 13.1 and Table 13.1 illustrate the above process and show the results obtained for the S (Shares) sector rank cross-correlations.

Figure 13.1 Cross-correlations over 46 years



red square actual results green diamond 5-yr central avs orange line 2017 averages (see section 6.10)

X-axis: Period 15 = average 46 years ending 30/9/05, 31/12/05, 31/3/06 and 30/6/06
Period 0 = average 46 years ending 30/9/20, 31/12/20, 31/3/21 and 30/6/21 (see section 6.8)

Table 13.1 Rank cross-correlations over 46 years

Actual and Five-year Central Averages

Period	S I	S Q	S P	S L	S H	S F	S G
15	.55	.61	.13	.22	.64	.22	.26
14	.57	.61	.14	.20	.67	.19	.23
13	.58	.61	.13	.20	.69	.20	.22
12	.60	.63	.16	.24	.71	.18	.19
11	.58	.62	.16	.26	.71	.16	.17
10	.60	.62	.15	.23	.72	.14	.16
9	.62	.62	.12	.20	.74	.10	.14
8	.62	.62	.11	.19	.74	.09	.13
7	.62	.61	.14	.20	.74	.09	.11
6	.61	.60	.15	.20	.74	.09	.12
5	.60	.57	.13	.21	.74	.07	.12
4	.61	.55	.12	.18	.76	.04	.10
3	.61	.54	.13	.18	.76	.03	.09
2	.61	.53	.13	.17	.76	.00	.08
1	.59	.52	.17	.14	.75	-.05	.04
0	.59	.51	.14	.16	.75	-.10	.06
0	*.59*	*.52*	*.15*	*.16*	*.75*	*-.06*	*.06*
-1	*.59*	*.51*	*.16*	*.16*	*.75*	*-.07*	*.06*
-2	*.59*	*.51*	*.16*	*.16*	*.75*	*-.08*	*.06*
-3	*.60*	*.51*	*.16*	*.16*	*.76*	*-.07*	*.05*
-4	*.61*	*.51*	*.15*	*.15*	*.77*	*-.05*	*.04*

S J	S C	S N	S B	S D	S W	S X	Period
.21	.14	.24	.12	.10	-.03	.01	15
.18	.12	.21	.10	.07	-.07	.01	14
.17	.12	.19	.09	.06	-.09	-.03	13
.20	.16	.18	.08	.07	-.10	-.04	12
.22	.15	.17	.08	.09	-.10	-.02	11
.19	.15	.14	.07	.07	-.07	-.01	10
.15	.17	.09	.09	.08	-.06	.01	9
.14	.13	.08	.06	.04	-.07	-.01	8
.16	.12	.08	.05	.05	-.08	-.01	7
.15	.13	.08	.07	.07	-.06	.00	6
.14	.16	.08	.10	.11	-.01	.03	5
.10	.15	.05	.09	.09	.00	.03	4
.09	.16	.04	.10	.10	.01	.03	3
.07	.16	.02	.10	.11	.01	.04	2
.01	.18	-.02	.13	.13	.07	.10	1
-.03	.22	-.01	.16	.15	.13	.16	0
.01	*.20*	*-.01*	*.14*	*.13*	*.08*	*.12*	*0*
-.01	*.21*	*-.01*	*.16*	*.14*	*.11*	*.14*	*-1*
-.01	*.22*	*-.02*	*.16*	*.15*	*.11*	*.15*	*-2*
.00	*.21*	*-.01*	*.16*	*.14*	*.10*	*.13*	*-3*
.01	*.20*	*.00*	*.14*	*.13*	*.07*	*.11*	*-4*

13.7 The five-year central average Period –2 results are the S sector cross-correlation assumptions that are tabulated in the first of the two half-page matrices in Table 16.3.

13.8 Table 16.3 contains both the **rank** cross-correlation assumptions and the **standard** cross-correlation assumptions (defined in paragraph 13.4). Both were calculated using the methodology described in paragraphs 13.2 and 13.3.

13.9 Sweeting (2007) states,

"... Correlation gives one measure of the linkages, but assumes that the relationships between the marginal distributions are constant, whatever the levels of those distributions. It is only appropriate if the marginal distributions are jointly elliptical. Other measures such as Kendall's tau and Spearman's rho, which do not depend on the marginal distributions, might be more appropriate. ...

If the degree of association is greater at extreme values of the marginal distributions (as it often is), then the above approaches understate the tail risk in the aggregate distribution. The solution is to use copulas. ... However, although there have been a number of papers on the subject of copulas, their use still appears to be limited. This is not least because the dependency structure is not straightforward. Since the dependency structure is largely defined by what happens at the tails, and as always, the tail of the distribution contains only a few observations, the form of the copula functions is not always clear."

13.10 Alcock and Hatherley (2009) demonstrate a means of incorporating asymmetrical structures during the portfolio process using copula functions. Specifically, they investigate how asymmetrical return dependencies affect the efficient frontier and subsequent portfolio performance under a dynamic rebalancing framework assuming normally distributed marginal returns.

13.11 To give some insight into the extent to which the degree of association may be greater at extreme values of the marginal distributions and when this might occur, Table 13.2 tabulates the **difference** between the standard cross-correlation assumptions and the rank cross-correlation assumptions.

Table 13.2 Standard less rank cross-correlations

	S	I	Q	P	L	H	F	G	J	C	N	B	D	W	X
S	0	.04	.09	-.02	.00	.00	.10	.04	.12	-.02	.08	.00	.01	-.08	.06
I	.04	0	.18	.05	.16	.07	.05	.12	.11	.14	.07	.13	.12	.07	.11
Q	.09	.18	0	.11	.04	.21	-.03	.01	.04	.03	-.03	.05	.09	-.05	.10
P	-.02	.05	.11	0	.00	.06	-.09	-.08	-.08	-.06	-.03	-.05	-.03	-.08	-.07
L	.00	.16	.04	.00	0	.12	.11	.12	.09	.11	.06	.10	.09	-.01	.09
H	.00	.07	.21	.06	.12	0	.09	.12	.12	.09	.10	.09	.07	-.03	.05
F	.10	.05	-.03	-.09	.11	.09	0	.09	.05	.09	.03	.05	.10	-.05	.07
G	.04	.12	.01	-.08	.12	.12	.09	0	.11	.05	.05	.00	.05	-.09	.05
J	.12	.11	.04	-.08	.09	.12	.05	.11	0	.09	.04	.08	.08	-.08	.06
C	-.02	.14	.03	-.06	.11	.09	.09	.05	.09	0	.04	.01	.02	-.03	.08
N	.08	.07	-.03	-.03	.06	.10	.03	.05	.04	.04	0	.00	.03	-.05	.04
B	.00	.13	.05	-.05	.10	.09	.05	.00	.08	.01	.00	0	.01	-.05	.06
D	.01	.12	.09	-.03	.09	.07	.10	.05	.08	.02	.03	.01	0	-.03	.06
W	-.08	.07	-.05	-.08	-.01	-.03	-.05	-.09	-.08	-.03	-.05	-.05	-.03	0	.01
X	.06	.11	.10	-.07	.09	.05	.07	.05	.06	.08	.04	.06	.06	.01	0

13.12 The green-shaded cells in the upper right triangular matrix of Table 13.2 are all the differences greater than or equal to 0.10. Twenty-two of the 105 cells are shaded - these indicate where the degree of association may be significant at extreme values of the marginal distributions.

13.13 The pink-shaded cell in the lower left triangular matrix of Table 13.2 is the combination where the difference is most negative. This indicates where the degree of dis-association might be significant at extreme values of the marginal distributions.

13.14 Eighty-two of the 105 cells in each triangular matrix are not shaded (in either triangle) - these indicate where the degree of association at the tails is likely to be similar to that for the central values.

13.15 It should be noted that all the correlations in this paper are annual. For example, the annual rank cross-correlation assumption for S against W is 0.11 and the annual standard cross-correlation assumption for S against W is 0.03. At first sight these low values seem to contradict the common actuarial assumption of positive long-term correlation between share returns and salary increases. However, for correlation periods longer than a year this might still hold.

14 Leads and Lags

14.1 Consider the correlation between D sector (Bond) forces and CPI forces for various lags, with blue shading to indicate the maximum cross-correlation in each row:

Table 14.1 D sector and CPI correlations

24 yr Correlation of D against CPI lagged by Y years:

Y / End date	0	6	6.25	6.5	6.75	7	7.25	7.5	7.75	8	8.25	8.5	8.75	9	10
30/6/84	.687	.791	.804	.810	**.816**	.813	.806	.801	.790	.771					
30/6/85	.591	.779	.790	.795	.805	.812	.813	**.815**	.812	.799	.790	.780	.766	.744	
30/6/86	.550	.818	**.820**	.814	.808	.804	.801	.801	.800	.794	.792	.791	.789	.776	.704
30/6/87	.513	.814	.828	.836	.848	**.852**	.843	.832	.815	.797	.790	.785	.783	.771	.729
30/6/88	.459	.809	.822	.830	.843	.853	.857	.861	**.862**	.852	.837	.820	.801	.778	.726
30/6/89	.414	.805	.818	.826	.838	.848	.852	.8571	**.857**	.851	.847	.844	.842	.827	.732
30/6/90	.347	.792	.817	.830	.841	.850	.853	**.858**	.856	.848	.843	.838	.837	.828	.790
30/6/91	.238	.732	.763	.792	.828	.852	.863	**.868**	.863	.850	.844	.838	.834	.824	.791
30/6/92	.124	.691	.726	.759	.797	.832	.857	.877	**.885**	.874	.858	.840	.825	.802	.764
30/6/93	.060	.627	.667	.705	.751	.792	.823	.853	.869	**.871**	.867	.858	.846	.814	.708
30/6/94	.081	.578	.607	.632	.666	.704	.739	.772	.794	.799	.801	.807	**.810**	.797	.685
30/6/95	.090	.547	.579	.606	.641	.675	.704	.732	.751	.756	.759	.763	**.770**	.758	.665
30/6/96	.080	.481	.523	.564	.609	.652	.685	.713	**.734**	.732	.726	.717	.711	.690	.587
30/6/97	.227	.450	.478	.507	.551	.593	.635	.678	.702	**.705**	.698	.686	.678	.647	.479
30/6/98	.441	.499	.521	.538	.563	.590	.610	.634	**.652**	.643	.636	.632	.624	.598	.429
30/6/99	.579	.580	.603	.618	.639	.656	.667	**.677**	.667	.645	.616	.589	.576	.537	.389
30/6/00	.675	.629	.653	.671	.694	.713	.723	**.732**	.726	.702	.672	.637	.604	.558	.367
30/6/01	.674	.657	.686	.710	.736	.756	.770	**.782**	.779	.761	.735	.705	.679	.634	.408
30/6/02	.703	.667	.693	.717	.748	.773	.791	**.805**	.804	.788	.767	.743	.721	.682	.471
30/6/03	.742	.718	.728	.739	.764	.784	.798	.809	**.810**	.800	.785	.768	.752	.719	.536
30/6/04	.765	.787	.783	.785	.794	.806	.816	.828	**.832**	.822	.804	.783	.771	.747	.610
30/6/05	.758	.802	.803	.809	.816	.820	.824	.832	**.835**	.830	.825	.824	.823	.806	.666
30/6/06	.713	.784	.791	.801	.807	.814	.818	.819	.824	.819	.815	.820	.828	**.829**	.773
30/6/07	.688	.750	.758	.772	.786	.799	.808	.812	.815	.815	.811	.811	**.817**	.814	.782
30/6/08	.726	.757	.759	.765	.769	.771	.780	.787	.795	.801	.797	.796	**.802**	.801	.756
30/6/09	.689	.742	.749	.765	.774	.781	**.7860**	.7855	.781	.766	.759	.755	.768	.778	.737
30/6/10	.638	.722	.730	.744	.758	.771	.785	.794	**.795**	.784	.766	.751	.744	.741	.711
30/6/11	.562	.695	.709	.730	.743	.753	.767	.775	**.779**	.774	.765	.758	.757	.758	.667
30/6/12	.491	.637	.660	.686	.711	.730	.748	.762	**.765**	.756	.746	.736	.738	.744	.677
30/6/13	.291	.576	.559	.577	.603	.637	.675	.699	.722	.728	.726	.730	**.731**	.729	.659
30/6/14	-.004	**.714**	.692	.652	.616	.580	.573	.592	.611	.632	.648	.662	.692	.712	.644
30/6/15	.045	.666	.666	.669	**.670**	.669	.655	.628	.600	.560	.538	.542	.573	.613	.604
30/6/16	.232	.588	.603	**.613**	.610	.591	.585	.584	.590	.587	.571	.548	.542	.542	.506
30/6/17	.300	.510	.535	.553	.560	.555	.559	**.562**	.559	.534	.511	.500	.511	.537	.450
30/6/18	.326	.427	.433	.450	.472	.478	.480	.477	.468	.451	.443	.443	.448	.458	**.487**
30/6/19	.221	.214	.267	.308	.339	.345	.335	.326	.317	.287	.267	.259	.277	.316	**.396**
30/6/20	**.332**	-.034	.007	.063	.103	.131	.166	.185	.180	.137	.090	.073	.098	.139	.252
30/6/21	**.605**	.028	-.003	-.041	-.055	-.053	-.029	-.001	.005	-.026	-.030	-.022	.001	.031	.118

14.2 Also consider the correlation between D sector forces and AWOTE forces for various lags:

Table 14.2 D sector and AWOTE correlations

24 yr Correlation of D against AWOTE lagged by Y years:

Y / End date	0	1	6.25	6.5	6.75	7	7.25	7.5	7.75	8	8.25	8.5	8.75	9	10
30/6/84	.527	.620	.743	.757	.786	.811	.826	**.835**	.822	.789					
30/6/85	.445	.554	.706	.727	.751	.779	.798	**.8059**	**.8061**	.789	.771	.756	.747	.728	
30/6/86	.330	.462	.673	.692	.716	.743	.766	**.777**	.773	.762	.748	.732	.737	.730	.690
30/6/87	.242	.360	.668	.692	.707	.729	.750	**.761**	.759	.750	.735	.719	.721	.716	.698
30/6/88	.169	.276	.649	.669	.695	.722	.742	**.758**	.753	.741	.728	.713	.718	.714	.688
30/6/89	.096	.217	.638	.653	.674	.700	.724	**.741**	.738	.730	.717	.711	.715	.711	.692
30/6/90	.003	.132	.622	.646	.682	.707	.723	**.737**	.722	.712	.695	.690	.703	.702	.683
30/6/91	-.085	.039	.565	.603	.647	.687	.722	**.746**	.742	.722	.691	.679	.681	.679	.670
30/6/92	-.147	-.023	.509	.553	.605	.649	.692	.726	.730	**.734**	.723	.710	.706	.672	.611
30/6/93	-.137	-.054	.470	.506	.561	.617	.667	.705	**.707**	.704	.689	.685	.695	.690	.571
30/6/94	-.057	.020	.468	.510	.550	.597	.644	.673	.677	**.683**	.673	.668	.673	.652	.571
30/6/95	-.008	.054	.453	.503	.554	.597	.635	**.669**	.661	.663	.652	.639	.643	.625	.522
30/6/96	.006	.126	.420	.465	.520	.581	.638	.686	**.688**	.684	.660	.649	.635	.607	.475
30/6/97	.130	.160	.414	.463	.520	.568	.621	.664	.669	**.680**	.672	.664	.662	.623	.446
30/6/98	.359	.258	.446	.483	.526	.568	.614	.656	.663	**.667**	.653	.642	.637	.608	.458
30/6/99	.516	.454	.517	.539	.566	.598	.637	**.665**	.660	.658	.640	.633	.632	.600	.456
30/6/00	.605	.583	.563	.587	.615	.641	.671	**.692**	.687	.683	.667	.653	.644	.610	.471
30/6/01	.614	.654	.598	.623	.651	.673	.701	**.724**	.721	.718	.700	.685	.676	.644	.496
30/6/02	.617	.675	.613	.639	.671	.703	.736	**.757**	.754	.747	.728	.716	.709	.679	.530
30/6/03	.620	.673	.643	.661	.686	.714	.745	.768	**.768**	.768	.755	.742	.735	.706	.577
30/6/04	.644	.663	.715	.702	.707	.728	.755	.778	.779	**.781**	.768	.757	.749	.723	.611
30/6/05	.656	.655	.777	.779	**.784**	.778	.776	.779	.776	.782	.776	.775	.779	.761	.636
30/6/06	.625	.645	.758	.764	.767	.768	.779	.783	.782	**.784**	.768	.767	.775	.770	.710
30/6/07	.616	.646	.761	.757	.764	.759	.766	.771	.768	.788	.791	.794	**.802**	.778	.711
30/6/08	.557	.630	.764	.762	.765	.758	.768	.764	.759	.775	.775	.779	**.785**	.772	.709
30/6/09	.518	.565	.783	.781	**.789**	.769	.771	.771	.760	.774	.781	.770	.772	.754	.686
30/6/10	.424	.538	.753	.769	.793	.792	.796	**.799**	.793	.792	.791	.793	.779	.755	.663
30/6/11	.388	.441	.729	.742	.763	.767	.774	.791	.801	.819	.823	**.826**	.815	.777	.663
30/6/12	.309	.405	.743	.753	.755	.757	.753	.765	.778	.802	.800	**.814**	**.815**	.794	.672
30/6/13	.099	.288	.739	.729	.683	.645	.641	.661	.703	.754	.778	**.781**	.764	.749	.707
30/6/14	-.172	.044	.605	.640	.651	.639	.604	.561	.539	.571	.617	.655	.671	**.673**	.649
30/6/15	-.122	-.160	.603	**.610**	.565	.485	.437	.438	.475	.526	.553	.547	.517	.488	.557
30/6/16	.152	-.060	.439	**.470**	.461	.413	.395	.369	.373	.377	.403	.447	.458	.454	.415
30/6/17	.349	.155	**.395**	.383	.339	.288	.260	.254	.280	.308	.349	.382	.374	.350	.395
30/6/18	**.458**	.357	.305	.326	.281	.249	.223	.172	.161	.172	.186	.237	.266	.271	.288
30/6/19	.502	**.524**	.114	.138	.142	.111	.083	.080	.077	.097	.112	.105	.082	.080	.228
30/6/20	**.533**	.568	-.049	-.046	-.100	-.116	-.120	-.114	-.063	-.043	-.027	.011	.003	.021	.020
30/6/21	.533	**.566**	.028	-.027	-.099	-.186	-.232	-.250	-.258	-.215	-.163	-.113	-.079	-.069	-.012

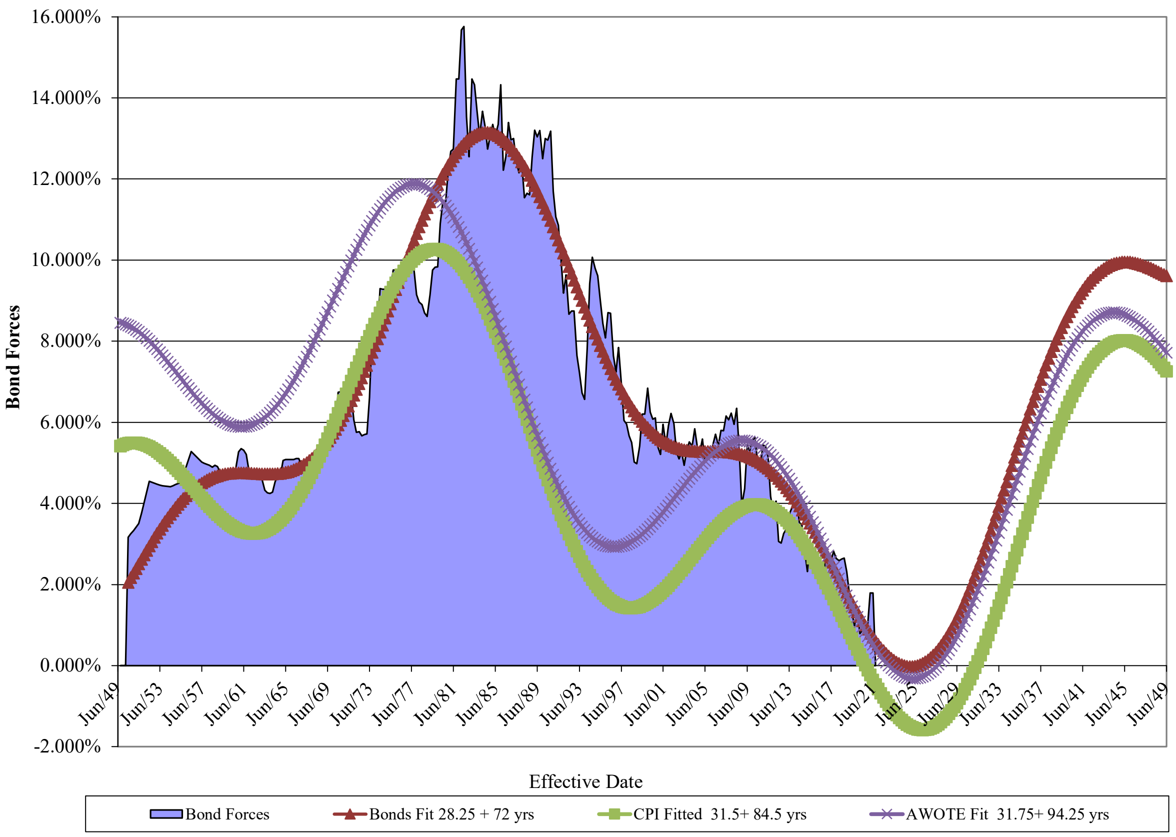
14.3 Tables 14.1 and 14.2 have some hidden columns but the hidden columns do not contain any maxima. Both tables are calculated from the main database, which contains annual forces at quarterly intervals (refer section 3.3).

14.4 Table 14.1 and 14.2 show until about 2005, that when CPI and AWOTE forces are lagged by about 7.5 to 8 years their correlation with D forces increases to a maximum. However since 2005, these relationships have gradually deteriorated and in the last 4 years the maxima have occurred at 0, 1 or 10 years.

14.5 It is difficult to find an economic justification as to why changes in CPI and AWOTE appeared to lead changes in bond rates by about 7.5 years with a correlation until 2005 of between 63% and 88%. Figure 14.1 below is based on the optimum Bond, CPI and AWOTE two-cycle curves described in section 5.10, 5.13 and 5.14. The actual Bond forces are also included on the same chart.

14.6 An examination of the CPI and AWOTE curves in Figure 14.1 visually shows that between about 1976 (when the two curves reached their maxima) and 1997 (when the two curves reached their second minima) both curves did in fact lead the Bond curve by about 7 to 8 years. However, outside this period very different relationships are evident. Interestingly, in 2025/2026 all three curves (Bonds, CPI and AWOTE) are projected to reach their third minima (and subsequently their next maxima) at almost the same times.

Figure 14.1 Bond, CPI and AWOTE - double cycles

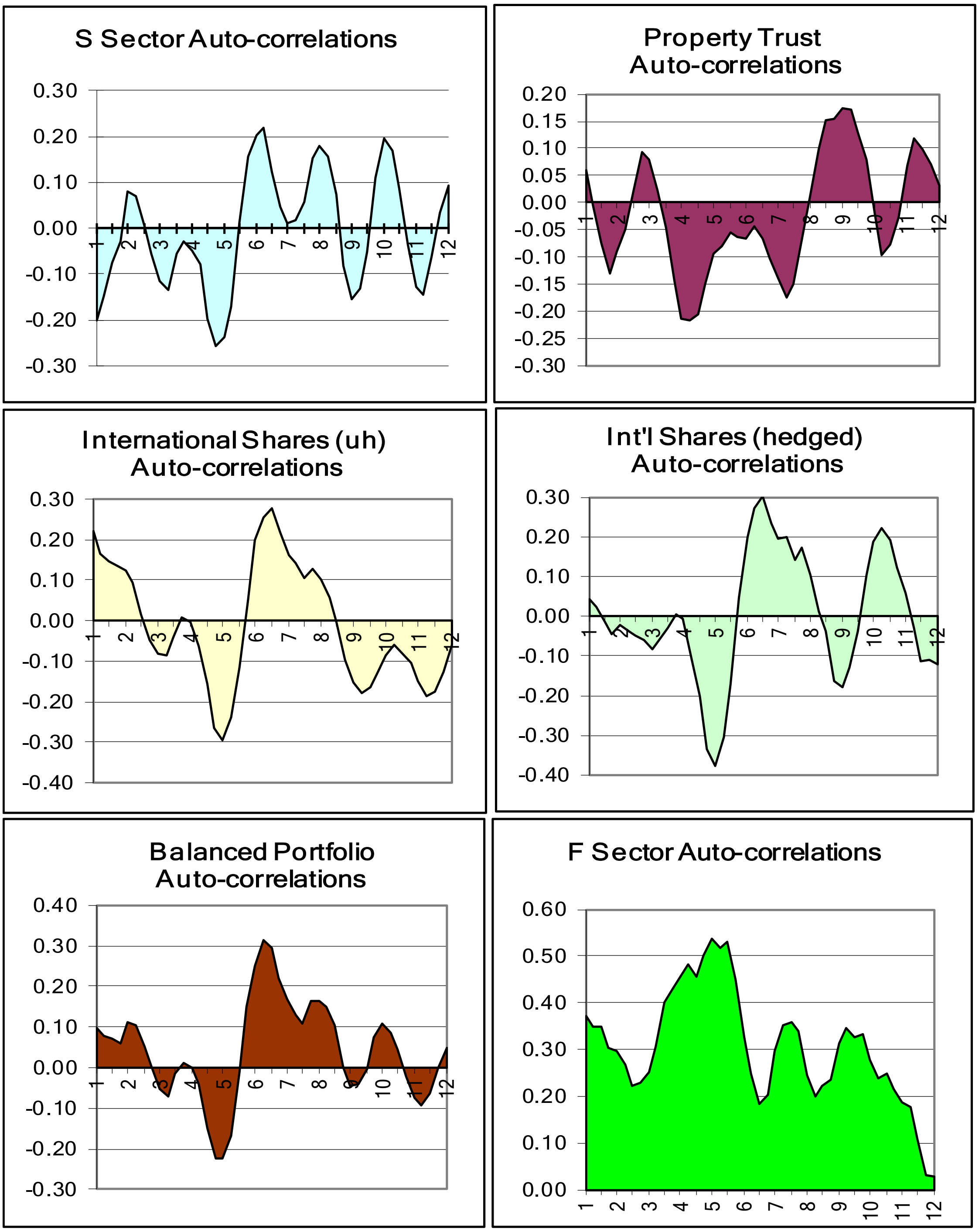


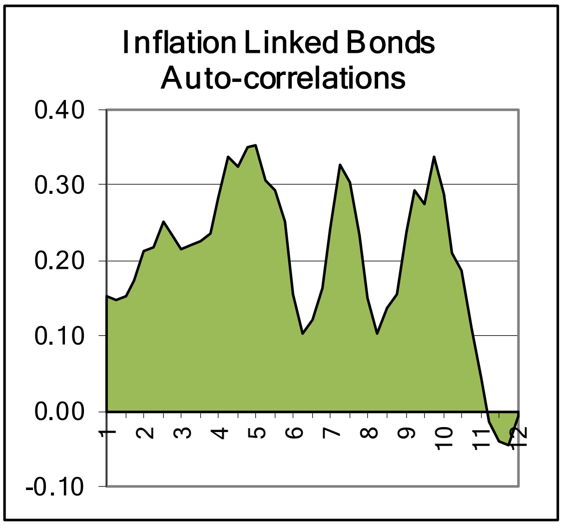
14.7 Tables 14.1 and 14.2 and Figure 14.1 all lead to the conclusion that, contrary to Grenfell and Sneddon (2017), it would be unsound to assume that D sector forces might lag CPI and AWOTE forces by 7 to 8 years in future, and particularly not in the immediate future.

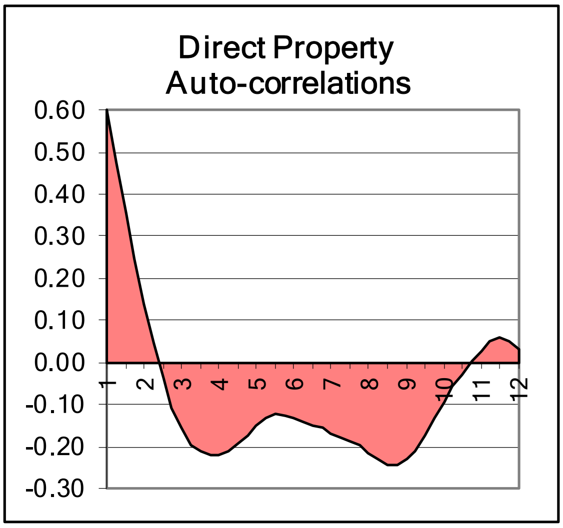
15 Auto-correlations

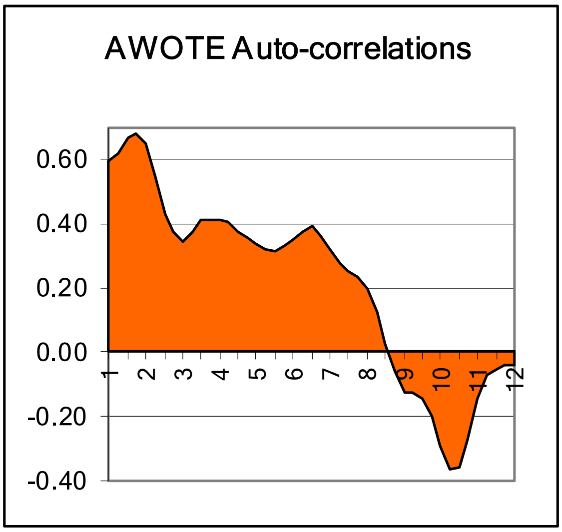
15.1 Auto-correlations vary significantly between the 15 sectors examined in this paper. To produce the following auto-correlation charts the annual forces for each sector were tabulated at quarterly intervals for the forty-year period from 30/6/1981 to 30/6/2021. From this forty-year or 160-quarter database, auto-correlations were calculated for lags of 4 to 48 quarters for eleven of the sectors plus the "Balanced" portfolio composite sector. Figure 15.1 below shows the results. The X-axis of these charts is the lag in years.

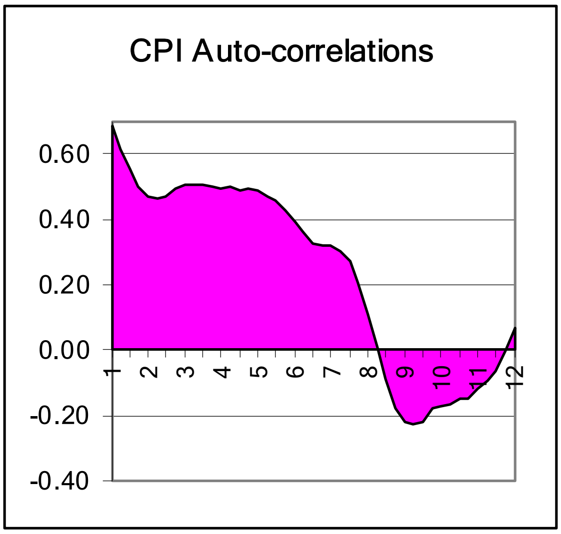
Figure 15.1 Auto-correlations over 40 years

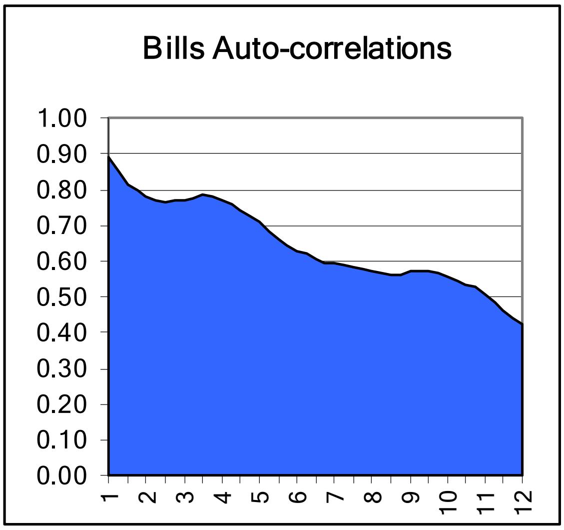


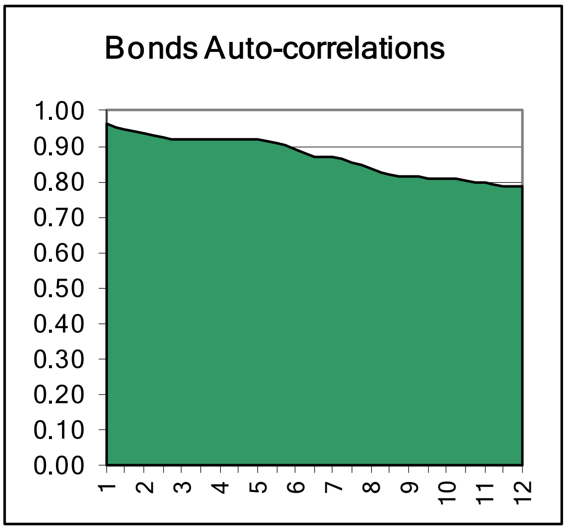

Inflation Linked Bonds
Auto-correlations
0.40
0.30
0.20
0.10
0.00
-0.10
1
2
3
4
5
6
7
8
9
10
11
12


Direct Property
Auto-correlations
0.60
0.50
0.40
0.30
0.20
0.10
0.00
-0.10
-0.20
-0.30
1
2
3
4
5
6
7
8
9
10
11
12


AWOTE Auto-correlations
0.60
0.40
0.20
0.00
-0.20
-0.40
1
2
3
4
5
6
7
8
9
10
11
12


CPI Auto-correlations
0.60
0.40
0.20
0.00
-0.20
-0.40
1
2
3
4
5
6
7
8
9
10
11
12


Bills Auto-correlations
1.00
0.90
0.80
0.70
0.60
0.50
0.40
0.30
0.20
0.10
0.00
1
2
3
4
5
6
7
8
9
10
11
12


Bonds Auto-correlations
1.00
0.90
0.80
0.70
0.60
0.50
0.40
0.30
0.20
0.10
0.00
1
2
3
4
5
6
7
8
9
10
11
12

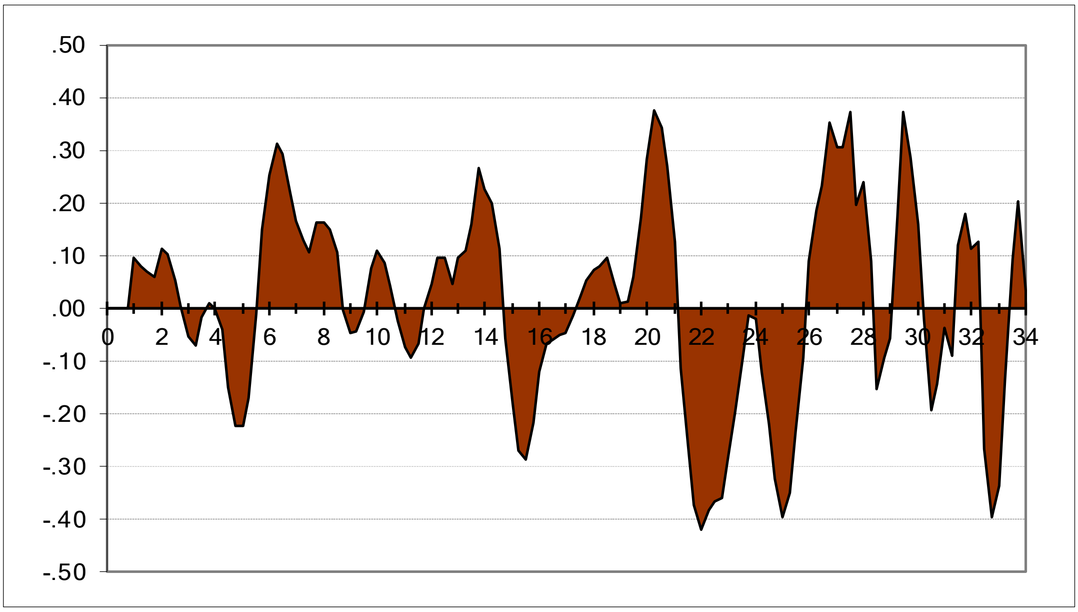
15.2 Table 15.1 below includes the above 40-year auto-correlations for the four "equity" sectors (Australian Shares, International Shares (unhedged), Property Trusts and International Shares (hedged)) for lags of one year to 5.5 years. Negative auto-correlations are shaded in pink – light pink for correlations greater than -0.10 and darker pink for correlations less than -0.10. Negative auto-correlations indicate that there is a tendency for market prices to "bounce back" after falls and/or to "drop again" after price increases. This tendency is somewhat similar, but different, to mean reversion. The S sector negative correlations are more significant than those for I, Q and H sectors.

Table 15.1 "Equity" sector auto-correlations

Lag	Sector			
(years)	S	I	Q	H
1.00	-.20	.22	.06	.04
1.25	-.15	.17	.00	.02
1.50	-.08	.15	-.07	-.01
1.75	-.03	.13	-.13	-.05
2.00	.08	.12	-.09	-.02
2.25	.07	.09	-.05	-.04
2.50	.00	.02	.02	-.05
2.75	-.05	-.05	.09	-.06
3.00	-.12	-.08	.08	-.08
3.25	-.13	-.08	.02	-.06
3.50	-.06	-.04	-.05	-.03
3.75	-.03	.01	-.15	.01
4.00	-.05	.00	-.21	-.01
4.25	-.08	-.06	-.22	-.11
4.50	-.20	-.16	-.20	-.20
4.75	-.26	-.26	-.15	-.34
5.00	-.24	-.30	-.09	-.38
5.25	-.17	-.24	-.08	-.30
5.50	.01	-.11	-.05	-.17

15.3 In Figure 15.2 below, the 40-year "Balanced" portfolio auto-correlations are shown, this time for lags from one year to 34 years (i.e. from 4 to 136 quarters).

Figure 15.2 Balanced portfolio auto-correlations over 40 years



15.4 For lags up to 2.75 years, the auto-correlations in Figure 15.2 are small but positive. For lags of 4.75 and 5 years the auto-correlations are both -0.22. Grenfell and Sneddon (2017) commented "Of the 19 observations up to a lag of 5.5 years, 12 (or 63%) are negative." However, after including the data for the last 4 years and after the changes to Balanced allocations explained in section 8.3, only 9 (or 47%) of these 19 observations are now negative. For lags beyond 5.5 years there is generally a slight positive bias in the results.

15.5 A key consideration is whether auto-correlations are stable or fluctuating over time. To judge this requires a lot of data. Asher (2006), when commenting on the analysis of auto-correlations in Grenfell (2005), stated:

"While there appears to be no theoretical justification – nor adequate data – for some of the longer-term correlations reported in that paper, the shorter-term cycles for share markets are more suggestive of some persistent relationships. In particular, the four-year cycle for equities does coincide with US presidential elections. ... This would also be consistent with other research, reported for instance in Cutler et al (1991) which shows that many markets overshoot in the short run and correct in the second (or perhaps fourth) year.

...The consensus opinion of experts is also of interest. There is no definitive answer here. ... Over a third [of financial and economics professors surveyed in 2000] thought that markets showed negative auto-correlations over a three to five-year horizon".

15.6 More data is now at hand. What does it indicate? Since the greatest contribution to Balanced (or Growth) portfolio auto-correlations comes from Australian shares, Grenfell and Sneddon (2017) analysed auto-correlations for the S sector over two <u>non-overlapping</u> 28-year periods, and for the D sector (10-year bonds) over two <u>non-overlapping</u> 33-year periods, described below:

S Sector:

Period	Average of four periods ending:
0	30/9/16, 31/12/16, 31/3/17 and 30/6/17 and
28	30/9/88, 31/12/88, 31/3/89 and 30/6/89

D sector:

Period	Average of four periods ending:
0	30/9/16, 31/12/16, 31/3/17 and 30/6/17 and
33	30/9/83, 31/12/83, 31/3/84 and 30/6/84

These periods were chosen because:

(a) all the available data was included except for the 3 months from 30 June 1959 to 30 September 1959,
(b) they provided at least 112 data observations for each "period", and
(c) 33 years for bonds was the mid-point of the enlarged (see section 5.5) database and also happened to be at or very close to the peak of the bond cycles.

15.7 The analysis described in section 15.6 above indicated that the 28-year average auto-correlations ending 1988/89 (for Shares) and the 33-year average auto-correlations ending 1983/84 (for Bonds) are similar, after re-scaling, to those ending 2016/17. This analysis has not been updated – though it is expected that similar conclusions would still apply if it were updated. The main reason for not updating is that section 15.12 below now demonstrates the stability of S and D sector auto-correlations over time in a more extensive, and hopefully effective, manner.

15.8 Auto-correlations are dependent on the period over which they are measured. For some distributions this applies even if the underlying distribution has been consistent over time. For example, if annual forces follow say an exact 35-year sine-curve over an infinite number of years, then auto-correlations measured over any say 30, 35 or 40-year periods will produce different auto-correlations for any given lag.

15.9 When the *Austmod* investment simulation model was first developed in the early 1990s it was based on 32-year projections partly because at that time less than 30 years' investment performance data was available for the main sectors. In view of the longer data period now available the model now produces 40-year projections and has recently been extended to also allow 100-year projections.

15.10 For these reasons Grenfell (2013) measured auto-correlations for shares and bonds over 40-year periods. However, a study of Figure 15.6 from that paper shows that there were two problems with the resulting bond auto-correlations:

(a) beyond a lag of about 14 years the curve was irregular (which was largely due to a lack of data points for the longer lags), and

(b) the blue ("central -2") curve crossed the X-axis at about 15.29 years implying a ridiculously long cycle of over 60 years.

Both these problems were overcome in Grenfell and Sneddon (2017) by measuring the bond auto-correlations over 60-year periods and using the data back to 30 June 1950 referred to in Section 5.5. For bonds the 60-year basis has been retained for the current work. Share auto-correlations cycles are much shorter than bond cycles so the S sector auto-correlations are still measured over 40-year periods.

15.11 The auto-correlation results were projected forward two years using the same methodology (and with the database extended forward by adding 6 years of "projected" annual forces for each quarter date) to that used for cross-correlations in sections 13.2 and 13.3.

15.12 Appendix B explains that the auto-correlations for Australian shares and 10-year bond rates "form two extremes". The following Figures 15.3 and 15.4 and Tables 15.2 and 15.3, therefore, illustrate the auto-correlations results for only the S and D sectors.

Figure 15.3 S sector auto-correlations over 40 years

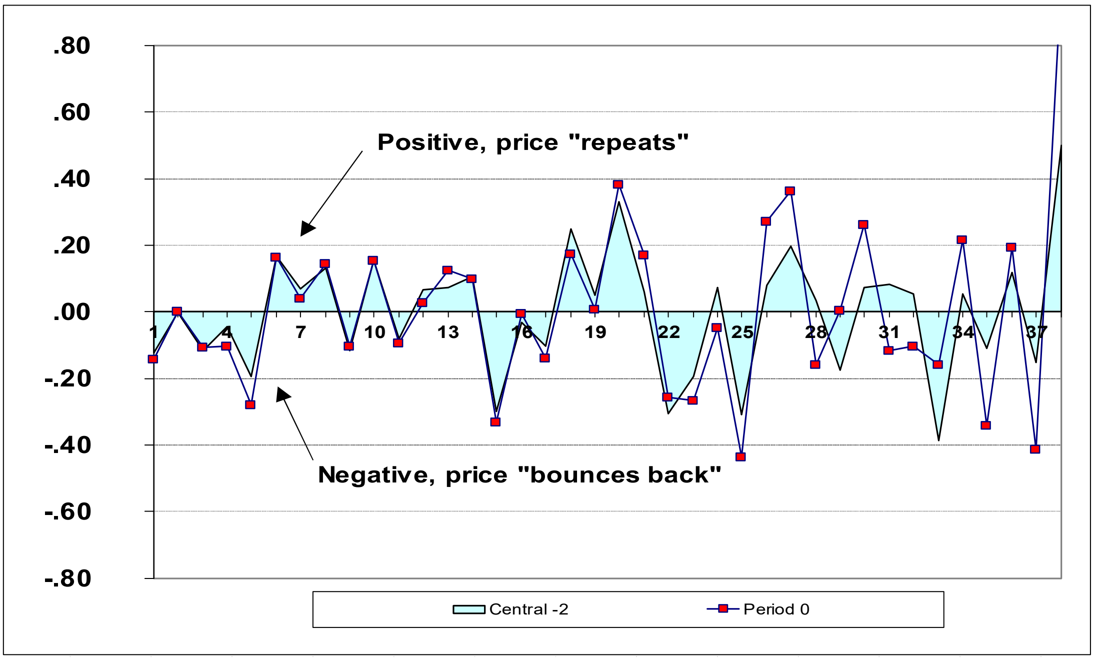


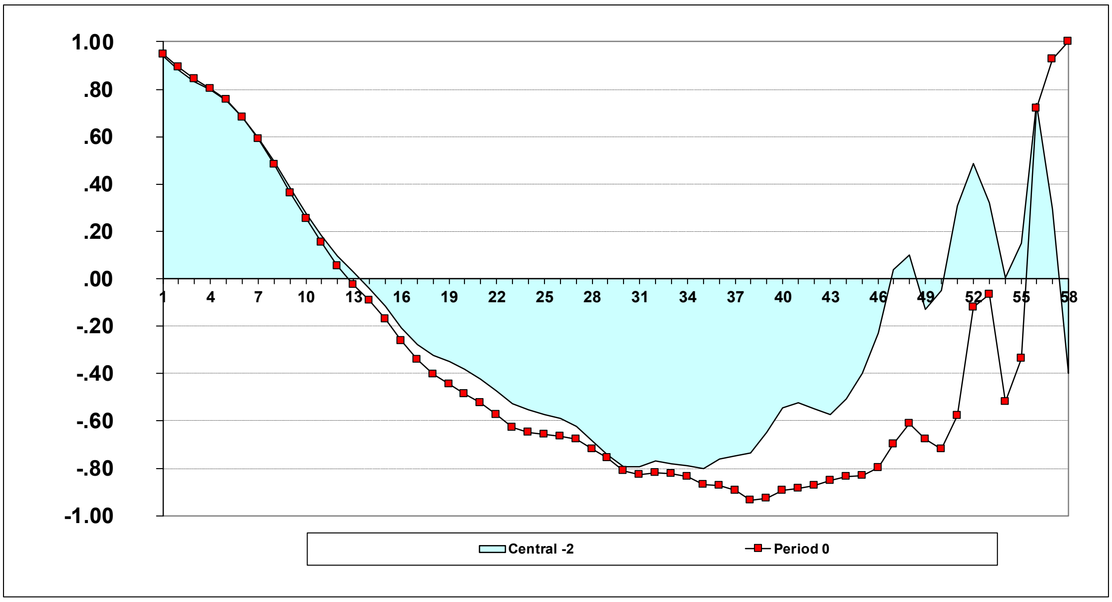
Table 15.2 S sector auto-correlations over 40 years

Period: Lag	21	20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0	-1	-2	-3	-4	-2 central
1	-.09	-.09	-.09	-.07	-.06	-.07	-.07	-.06	-.07	-.04	-.08	-.08	-.06	-.06	-.14	-.09	-.08	-.08	-.08	-.11	-.19	-.14	-.14	-.12	-.11	-.11	-.12
2	-.17	-.18	-.18	-.18	-.17	-.19	-.17	-.16	-.16	-.17	-.17	-.13	-.14	-.12	-.09	-.10	-.09	-.09	-.09	-.12	-.03	.00	.05	.03	-.03	-.06	.00
3	.00	.01	.01	-.01	-.01	.01	-.01	-.01	-.01	-.05	-.04	-.08	-.02	-.03	-.05	-.06	-.09	-.08	-.07	-.03	-.03	-.11	-.09	-.08	-.15	-.16	-.12
4	.11	.09	.09	.08	.07	.07	.04	.02	.01	-.04	-.05	-.09	-.07	-.09	-.09	-.05	-.08	-.07	-.08	-.07	-.11	-.10	-.02	-.02	-.04	-.04	-.05
5	-.06	-.08	-.09	-.08	-.08	-.09	-.12	-.13	-.10	-.11	-.13	-.12	-.12	-.12	-.11	-.05	-.09	-.12	-.13	-.17	-.19	-.28	-.18	-.17	-.18	-.17	-.19
6	-.12	-.12	-.11	-.11	-.10	-.12	-.12	-.12	-.06	.04	.06	.04	.01	.01	.11	.22	.27	.27	.29	.29	.23	.17	.18	.18	.16	.15	.17
7	.01	.03	.04	.03	.04	.05	.06	.05	.10	.13	.10	.10	.07	.09	.17	.10	.18	.20	.20	.18	.07	.04	.04	.05	.10	.11	.07
8	.32	.33	.31	.32	.32	.32	.30	.30	.28	.26	.25	.28	.26	.27	.19	.16	.14	.11	.12	.09	.11	.14	.17	.14	.11	.10	.13
9	-.06	-.08	-.08	-.10	-.11	-.16	-.23	-.23	-.23	-.21	-.21	-.17	-.13	-.14	-.17	-.08	-.09	-.07	-.05	-.08	-.07	-.10	-.14	-.12	-.11	-.11	-.12
10	-.11	-.13	-.13	-.11	-.09	-.15	-.11	-.12	-.09	-.08	-.06	-.02	.04	.05	.11	.09	.03	.07	.10	.12	.09	.16	.18	.18	.13	.13	.15
11	-.03	-.02	-.01	-.04	-.01	.03	.00	.00	-.03	-.02	.00	-.05	-.06	-.09	-.10	-.01	-.09	-.11	-.13	-.14	-.06	-.09	-.11	-.09	-.06	-.06	-.08
12	.03	.02	-.01	.00	-.02	-.05	-.04	-.04	-.11	-.12	-.17	-.18	-.19	-.18	-.11	-.01	-.04	-.02	-.02	.00	.01	.03	.10	.08	.06	.06	.07
13	-.03	-.05	-.07	-.06	-.05	-.05	-.04	-.02	-.05	-.05	-.06	-.02	-.03	-.03	.10	.17	.23	.24	.23	.22	.24	.12	.06	.06	.05	.06	.07
14	-.10	-.15	-.14	-.13	-.12	-.12	-.07	-.06	.00	.08	.11	.11	.11	.12	.16	.14	.14	.11	.12	.13	.05	.10	.14	.11	.10	.09	.11
15	-.14	-.15	-.19	-.18	-.19	-.15	-.12	-.15	-.14	-.21	-.23	-.20	-.24	-.29	-.33	-.33	-.29	-.28	-.28	-.33	-.31	-.33	-.31	-.29	-.29	-.28	-.30
16	.10	.08	.03	-.03	-.06	-.03	-.11	-.13	-.15	-.08	-.03	.04	.07	.09	.11	.04	.02	.01	.03	.04	.00	-.01	-.02	-.03	-.04	-.07	-.03
17	.11	.09	.09	.04	.03	-.07	-.12	-.11	-.10	-.13	-.11	-.12	-.10	-.11	-.17	-.13	-.10	-.09	-.10	-.11	-.12	-.14	-.10	-.09	-.10	-.09	-.10
18	-.06	-.05	-.06	-.02	.03	.07	.14	.13	.09	.18	.19	.20	.17	.18	.23	.21	.18	.16	.16	.16	.19	.17	.23	.26	.29	.29	.25
19	-.06	.02	.03	-.06	-.04	.02	.05	.07	-.01	-.02	-.05	-.04	-.03	-.03	-.07	.03	.05	.06	.07	.06	.05	.01	.01	.02	.11	.11	.05
20	.00	.02	.06	.07	.03	.05	.16	.26	.31	.30	.31	.30	.29	.28	.40	.38	.38	.39	.41	.41	.40	.38	.35	.30	.30	.31	.33
21	.02	-.10	-.16	.02	.03	.14	.23	.29	.23	.15	.13	.16	.19	.19	.13	.17	.18	.17	.17	.15	.14	.17	.07	.07	-.01	.00	.06
22	-.04	-.07	-.16	-.29	-.37	-.34	-.24	-.24	-.27	-.35	-.35	-.36	-.36	-.38	-.37	-.37	-.40	-.40	-.39	-.40	-.35	-.26	-.25	-.24	-.38	-.39	-.30
23	-.07	-.06	-.11	-.23	-.32	-.45	-.49	-.43	-.41	-.49	-.42	-.38	-.34	-.34	-.34	-.34	-.32	-.30	-.29	-.29	-.22	-.27	-.18	-.14	-.21	-.18	-.20
24	.12	.13	.12	.03	.11	.21	.09	.07	.04	.06	.15	.10	.08	.08	.08	.08	.06	.04	.02	.06	.01	-.05	.10	.13	.11	.07	.07
25	-.01	.01	.03	.02	.08	.23	.26	.18	.15	-.09	-.12	-.12	-.18	-.20	-.22	-.19	-.16	-.18	-.22	-.23	-.32	-.44	-.36	-.33	-.22	-.19	-.31
26	-.19	-.08	-.13	-.15	-.17	-.11	.09	.12	.15	.22	.27	.30	.24	.29	.38	.45	.49	.43	.44	.43	.36	.27	.17	.10	-.05	-.09	.08
27	-.02	-.09	-.02	-.19	-.13	-.10	-.02	.10	.14	.32	.30	.27	.25	.27	.33	.30	.39	.41	.44	.42	.35	.36	.06	.15	.22	.19	.20
28	.03	.09	.23	.42	.40	.37	.45	.50	.28	-.08	-.16	-.13	-.19	-.19	-.23	-.47	-.44	-.42	-.39	-.42	-.35	-.16	.04	-.03	.14	.17	.03
29	.05	-.01	-.10	.21	.18	.37	.32	.32	.21	-.20	-.07	-.03	.00	.04	-.09	-.06	-.14	-.10	-.09	-.12	.16	.00	-.06	-.20	-.31	-.31	-.18
30	.26	.27	.24	.02	-.21	-.44	-.17	-.19	-.20	-.36	-.17	-.18	-.06	-.05	.03	.15	.03	.14	.17	.29	.21	.26	.03	.08	.02	-.02	.07
31	-.32	-.27	-.23	-.21	-.10	-.28	-.45	-.31	-.28	-.26	-.14	-.16	-.23	-.28	-.20	-.02	-.07	-.06	-.34	-.33	-.26	-.12	.06	.17	.15	.15	.08
32	.12	.06	.21	.34	.35	.46	.13	.00	-.09	-.11	-.09	-.17	-.31	-.25	-.17	-.05	-.14	-.43	-.41	-.38	-.09	-.10	.10	.02	.13	.12	.05
33	.12	.09	-.10	.22	.07	.16	.24	.05	.03	-.35	-.37	-.35	-.54	-.40	-.11	-.14	.20	.38	.34	.55	.26	-.16	-.29	-.43	-.53	-.54	-.39
34	.09	-.19	-.54	-.53	-.72	-.76	-.57	-.46	-.08	.63	.63	.68	.64	.70	.81	.52	.61	.48	.45	.51	.17	.22	-.01	.05	.07	-.07	.05
35	-.35	.04	.01	-.52	-.52	-.54	-.68	-.71	-.12	.55	.31	.28	.19	.15	-.20	.14	.00	-.52	-.46	-.63	-.60	-.34	-.06	-.09	-.12	.06	-.11
36	.14	.51	.56	.32	.41	.52	.16	-.50	-.42	-.73	-.75	-.59	-.58	-.46	-.42	-.82	-.83	-.49	-.21	-.12	.25	.19	.35	.44	.26	-.64	.12
37	.41	-.33	-.11	.74	.66	.89	.70	.70	.36	-.28	-.06	-.15	.58	.77	.81	.50	-.77	.39	.50	.27	-.46	-.41	-.40	-.43	.87	-.40	-.15
38	.00	.00	.00	.50	.00	1.0	.00	.50	.00	1.0	.50	.00	1.0	-.50	-.50	.50	-.50	1.0	.00	.50	.50	1.0	.50	-.50	1.0	.50	.50

Period 0: average 40 years ending 30/9/20, 31/12/20, 31/3/21 and 30/6/21
Period 21: average 40 years ending 30/9/99, 31/12/99, 31/3/00 and 30/6/00 (see section 6.8)

For Shares, the green shading in Figure 15.1 marks the first tabulated occurrence after lag 2 years of auto-correlations in the range -1% to +1%. The pink shading is the maximum auto-correlations between lags 7 and 10 years and the minimum auto-correlations between lags 10 and 21 years. The three rows (or double rows) of shading, particularly the third, indicate significant stability in S sector auto-correlations over the entire 62-year period.

Figure 15.4 Bond auto-correlations over 60 years



For Bonds, up to year 35, the “central -2” curve and the “period 0” curve are within .10 of each other, again indicating stability of results over time. After year 6, the “central -2” curve lies to the right of the “period 0” curve, indicating that the longer lag Bond auto-correlation features (up to at least 30-year lags) are now occurring slower.

Table 15.3 Bond auto-correlations over 60 years

Period: / Lag	11	10	9	8	7	6	5	4	3	2	1	0	-1	-2	-3	-4	-2 central
1	.94	.94	.94	.94	.94	.94	.94	.94	.94	.94	.95	.95	.94	.94	.94	.94	.94
2	.88	.88	.88	.87	.87	.88	.88	.88	.88	.89	.89	.89	.88	.88	.88	.88	.88
3	.82	.82	.82	.82	.82	.82	.82	.82	.83	.83	.84	.84	.84	.83	.83	.83	.83
4	.79	.78	.78	.78	.78	.77	.77	.78	.78	.79	.80	.80	.80	.80	.79	.79	.80
5	.75	.74	.74	.74	.73	.73	.73	.73	.74	.74	.75	.76	.75	.75	.75	.75	.75
6	.67	.66	.66	.66	.65	.65	.65	.65	.65	.66	.67	.68	.68	.68	.68	.68	.68
7	.57	.57	.57	.56	.56	.56	.56	.56	.56	.56	.58	.59	.59	.59	.60	.61	.60
8	.47	.46	.45	.45	.45	.45	.44	.45	.45	.45	.46	.48	.49	.49	.50	.51	.49
9	.35	.34	.33	.33	.33	.33	.33	.33	.33	.34	.35	.36	.37	.38	.39	.40	.38
10	.23	.22	.21	.21	.20	.20	.21	.21	.21	.22	.24	.25	.26	.27	.28	.31	.28
11	.11	.10	.08	.08	.08	.08	.08	.10	.10	.11	.13	.15	.16	.17	.20	.23	.18
12	.00	-.01	-.03	-.04	-.04	-.04	-.03	-.02	.00	.02	.03	.05	.07	.09	.12	.15	.10
13	-.09	-.12	-.13	-.14	-.14	-.14	-.13	-.11	-.09	-.07	-.05	-.02	.00	.02	.05	.09	.03
14	-.20	-.24	-.26	-.25	-.26	-.26	-.24	-.22	-.19	-.17	-.14	-.09	-.07	-.04	-.01	.01	-.04
15	-.30	-.34	-.38	-.38	-.38	-.38	-.36	-.33	-.31	-.27	-.23	-.17	-.14	-.12	-.09	-.06	-.12
16	-.40	-.45	-.50	-.52	-.53	-.50	-.48	-.46	-.43	-.38	-.32	-.26	-.23	-.21	-.19	-.14	-.21
17	-.51	-.55	-.60	-.62	-.63	-.62	-.60	-.56	-.52	-.47	-.41	-.34	-.32	-.29	-.25	-.19	-.28
18	-.57	-.63	-.66	-.68	-.69	-.71	-.69	-.62	-.57	-.53	-.46	-.40	-.38	-.34	-.28	-.22	-.32
19	-.63	-.68	-.71	-.71	-.74	-.75	-.72	-.67	-.61	-.56	-.51	-.45	-.41	-.36	-.30	-.23	-.35
20	-.66	-.71	-.75	-.78	-.77	-.77	-.75	-.72	-.67	-.62	-.56	-.49	-.44	-.39	-.33	-.26	-.38
21	-.69	-.74	-.81	-.83	-.82	-.79	-.77	-.75	-.73	-.69	-.60	-.52	-.48	-.43	-.37	-.30	-.42
22	-.73	-.80	-.84	-.86	-.86	-.85	-.80	-.79	-.77	-.74	-.67	-.57	-.53	-.49	-.42	-.36	-.48
23	-.76	-.81	-.83	-.85	-.86	-.85	-.84	-.80	-.78	-.75	-.70	-.63	-.58	-.53	-.48	-.41	-.53
24	-.75	-.78	-.80	-.81	-.82	-.84	-.83	-.81	-.76	-.74	-.70	-.65	-.61	-.56	-.50	-.44	-.55
25	-.72	-.74	-.76	-.78	-.79	-.81	-.81	-.80	-.78	-.73	-.70	-.66	-.63	-.59	-.53	-.46	-.57
26	-.70	-.72	-.74	-.76	-.78	-.78	-.79	-.80	-.79	-.77	-.70	-.66	-.64	-.61	-.56	-.48	-.59
27	-.70	-.71	-.74	-.76	-.77	-.77	-.78	-.79	-.80	-.79	-.76	-.68	-.66	-.64	-.60	-.54	-.62
28	-.68	-.69	-.71	-.73	-.74	-.75	-.76	-.77	-.78	-.79	-.77	-.72	-.71	-.70	-.67	-.63	-.69
29	-.68	-.68	-.67	-.69	-.70	-.72	-.73	-.75	-.76	-.77	-.77	-.76	-.77	-.75	-.73	-.71	-.74
30	-.67	-.66	-.65	-.66	-.68	-.69	-.71	-.74	-.75	-.76	-.78	-.81	-.81	-.80	-.78	-.77	-.80

Period 0: average 60 years ending 30/9/20, 31/12/20, 31/3/21 and 30/6/21
Period 11: average 60 years ending 30/9/09, 31/12/09, 31/3/10 and 30/6/10 (see section 6.8)

For Bonds, where the curve for lags 0 to at least lag 11 is positive, the green shading is an indication of one-quarter of a full cycle. It marks the first tabulated occurrence of auto-correlations in the range -1% to +1%. It has been reasonably stable over time.

15.13 Table 15.2 includes only the first 30-year lags because near to and beyond the half-way point the auto-correlations often fluctuate.

16 Assumptions

16.1 The following assumptions, developed as explained in previous sections, are gross of (i.e. before) tax and fees and are before additions for imputation credits (ICs). They have not yet been "road-tested", but are not significantly different from the previously tested ("OLD") assumptions.

Table 16.1 Investment assumptions

Sector		Risk margin (arithmetic average)	Mean rate (arithmetic average)	Compound average	Coefficient of variation	Standard deviation of rates	Skewness	Kurtosis
							'Moderate'	
							High'	
							'Extreme'	
S	Shares	5.1%	10.1%	9.0%	1.535	15.5%	-16%	109%
I	Int'l Shrs	4.1%	9.1%	8.2%	1.539	14.0%	-60%	185%
Q	Prop Trust	4.1%	9.1%	8.0%	1.539	14.0%	-233%	963%
P	Direct Prop	2.3%	7.3%	7.1%	0.822	6.0%	-134%	259%
H	Hedged IS	4.3%	9.3%	8.5%	1.398	13.0%	-77%	185%
L	Loans/credit	1.7%	6.7%	6.6%	0.523	3.5%	78%	-16%
F	Fixed Int	1.0%	6.0%	5.9%	0.750	4.5%	57%	-2%
G	Semi-govt	0.9%	5.9%	5.8%	0.644	3.8%	67%	-81%
J	Int'l Fxd Int	0.9%	5.9%	5.8%	0.627	3.7%	36%	18%
C	Cash	-0.3%	4.7%	4.7%	0.618	2.9%	48%	-73%
N	Infln Linked	1.5%	6.5%	6.4%	0.830	5.4%	5%	5%
Balncd	Balanced	3.03%	8.03%	7.69%	1.068	8.57%	-77%	238%
CapStb	Cap Stable	1.43%	6.43%	6.34%	0.654	4.20%	5%	135%
B	Bills	-0.30%	4.70%	4.66%	0.639	3.00%	60%	-46%
D	Bonds		5.00%	4.97%	0.480	2.40%	11%	-106%
W	AWOTE	-2.00%	3.00%	2.98%	0.700	2.10%	192%	429%
X	CPI	-3.00%	2.00%	1.99%	0.800	1.60%	98%	-10%

Table 16.2 Auto-correlation assumptions

Lag (years)	S sector	D sector	Lag (years)	S sector	D sector	Lag (years)	D sector
1	-.12	.94	21	.06	-.42	41	-.52
2	.00	.88	22	-.30	-.48	42	-.55
3	-.12	.83	23	-.20	-.53	43	-.57
4	-.05	.80	24	.07	-.55	44	-.51
5	-.19	.75	25	-.31	-.57	45	-.40
6	.17	.68	26	.08	-.59	46	-.23
7	.07	.60	27	.20	-.62	47	.04
8	.13	.49	28	.03	-.69	48	.10
9	-.12	.38	29	-.18	-.74	49	-.13
10	.15	.28	30	.07	-.80	50	-.05
11	-.08	.18	31	.08	-.79	51	.31
12	.07	.10	32	.05	-.77	52	.49
13	.07	.03	33	-.39	-.78	53	.32
14	.11	-.04	34	.05	-.79	54	.00
15	-.30	-.12	35	-.11	-.80	55	.15
16	-.03	-.21	36	.12	-.76	56	.74
17	-.10	-.28	37	-.15	-.75	57	.30
18	.25	-.32	38	.50	-.74	58	-.40
19	.05	-.35	39		-.65		
20	.33	-.38	40		-.54		

Table 16.3 Cross-correlation assumptions

RANK CROSS-CORRELATIONS @ 2 YEARS (5-point average, rounded)

	S	I	Q	P	L	H	F	G	J	C	N	B	D	W	X
S	1	.59	.51	.16	.16	.75	-.08	.06	-.01	.22	-.02	.16	.15	.11	.15
I	.59	1	.37	.18	.07	.82	.09	.02	.08	.07	.13	.04	.05	-.07	.04
Q	.51	.37	1	.19	.24	.33	.23	.11	.30	.16	.28	.07	.07	.09	.01
P	.16	.18	.19	1	.31	.16	-.02	.24	.09	.46	.07	.42	.33	.44	.59
L	.16	.07	.24	.31	1	.09	.57	.79	.64	.80	.61	.77	.80	.60	.60
H	.75	.82	.33	.16	.09	1	-.04	-.02	.03	.10	.02	.07	.09	.00	.08
F	-.08	.09	.23	-.02	.57	-.04	1	.72	.87	.47	.87	.48	.46	.25	.18
G	.06	.02	.11	.24	.79	-.02	.72	1	.67	.87	.69	.90	.85	.66	.61
J	-.01	.08	.30	.09	.64	.03	.87	.67	1	.52	.82	.49	.51	.35	.27
C	.22	.07	.16	.46	.80	.10	.47	.87	.52	1	.51	.97	.94	.73	.73
N	-.02	.13	.28	.07	.61	.02	.87	.69	.82	.51	1	.52	.50	.30	.32
B	.16	.04	.07	.42	.77	.07	.48	.90	.49	.97	.52	1	.94	.73	.74
D	.15	.05	.07	.33	.80	.09	.46	.85	.51	.94	.50	.94	1	.70	.69
W	.11	-.07	.09	.44	.60	.00	.25	.66	.35	.73	.30	.73	.70	1	.78
X	.15	.04	.01	.59	.60	.08	.18	.61	.27	.73	.32	.74	.69	.78	1
	Average	.414933													

STANDARD CROSS-CORRELATIONS @ 2 YEARS (5-point average, rounded)

	S	I	Q	P	L	H	F	G	J	C	N	B	D	W	X
S	1	.63	.60	.14	.16	.75	.02	.10	.11	.20	.06	.16	.16	.03	.21
I	.63	1	.55	.23	.23	.89	.14	.14	.19	.21	.20	.17	.17	.00	.15
Q	.60	.55	1	.30	.28	.54	.20	.12	.34	.19	.25	.12	.16	.04	.11
P	.14	.23	.30	1	.31	.22	-.11	.16	.01	.40	.04	.37	.30	.36	.52
L	.16	.23	.28	.31	1	.21	.68	.91	.73	.91	.67	.87	.89	.59	.69
H	.75	.89	.54	.22	.21	1	.05	.10	.15	.19	.12	.16	.16	-.03	.13
F	.02	.14	.20	-.11	.68	.05	1	.81	.92	.56	.90	.53	.56	.20	.25
G	.10	.14	.12	.16	.91	.10	.81	1	.78	.92	.74	.90	.90	.57	.66
J	.11	.19	.34	.01	.73	.15	.92	.78	1	.61	.86	.57	.59	.27	.33
C	.20	.21	.19	.40	.91	.19	.56	.92	.61	1	.55	.98	.96	.70	.81
N	.06	.20	.25	.04	.67	.12	.90	.74	.86	.55	1	.52	.53	.25	.36
B	.16	.17	.12	.37	.87	.16	.53	.90	.57	.98	.52	1	.95	.68	.80
D	.16	.17	.16	.30	.89	.16	.56	.90	.59	.96	.53	.95	1	.67	.75
W	.03	.00	.04	.36	.59	-.03	.20	.57	.27	.70	.25	.68	.67	1	.79
X	.21	.15	.11	.52	.69	.13	.25	.66	.33	.81	.36	.80	.75	.79	1
	Average	.455467													

17 Assumptions Gross/Net of Tax

17.1 All results in previous sections are gross of (i.e. before) tax and imputation credits (ICs). The following results illustrate the impact of tax and imputation credits on the Section 16 assumptions for mean rates, for superannuation in the accumulation and in the pension stage.

Table 17.1 Gross/net of tax

Sector		Mean rate (arithmetic average)			Accum-ulation	Pension
		Before tax	After tax and imputation credits	Average tax rate	Compound average rates after tax and imputation credits	
S	Shares	10.10%	10.08%	0.2%	9.21%	10.49%
I	Int'l Shrs	9.10%	8.33%	8.4%	7.60%	8.55%
Q	Prop Trust	9.10%	8.14%	10.5%	7.23%	8.30%
P	Direct Prop	7.30%	6.23%	14.7%	6.09%	7.12%
H	Hedged IS	9.30%	8.51%	8.5%	7.87%	8.86%
L	Loans/credit	6.70%	5.70%	15.0%	5.65%	6.64%
F	Fixed Int	6.00%	5.10%	15.0%	5.02%	5.91%
G	Semi-govt	5.90%	5.02%	15.0%	4.96%	5.83%
J	Int'l Fxd Int	5.90%	5.02%	15.0%	4.96%	5.84%
C	Cash	4.70%	4.00%	15.0%	3.97%	4.66%
N	Infln Linked	6.50%	5.58%	14.2%	5.47%	6.36%
Balncd	Balanced	8.03%	7.51%	6.4%	7.24%	8.27%
CapStb	Cap Stable	6.43%	5.72%	10.9%	5.66%	6.55%
B	Bills	4.70%	4.00%	15.0%	3.96%	4.66%
D	Bonds	5.00%	4.25%	15.0%	4.23%	4.97%

17.2 In the accumulation stage, the above table allows for income tax at the 15% superannuation rate and, on an approximate basis, for imputation credits and the lower rates of tax on realised capital gains after 12 months.

17.3 In the pension stage, the final column of the above table allows for imputation credits (but no income or capital gains taxes).

18 Assumptions Gross/Net of Fees

18.1 All results in previous sections are gross of (i.e. before) fees. The following results illustrate the impact of wholesale passive investment fees on the Section 16 and 17 assumptions.

Table 18.1 Gross/net of fees

Sector		Mean rate (arithmetic average)			Accum-ulation	Pension
		Before tax Before fees	Before tax After fees	After tax & IC's After fees	Compound average rates after tax & ICs after fees	
S	Shares	10.10%	9.84%	9.86%	8.99%	10.22%
I	Int'l Shrs	9.10%	8.81%	8.09%	7.36%	8.26%
Q	Prop Trust	9.10%	8.81%	7.90%	6.98%	8.00%
P	Direct Prop	7.30%	6.60%	5.63%	5.49%	6.42%
H	Hedged IS	9.30%	9.01%	8.27%	7.63%	8.57%
L	Loans	6.70%	6.41%	5.45%	5.41%	6.35%
F	Fixed Int	6.00%	5.82%	4.95%	4.87%	5.73%
G	Semi-govt	5.90%	5.72%	4.86%	4.81%	5.65%
J	Int'l Fxd Int	5.90%	5.72%	4.86%	4.81%	5.66%
C	Cash	4.70%	4.55%	3.87%	3.84%	4.51%
N	Infln Linked	6.50%	6.31%	5.41%	5.31%	6.17%
Balncd	Balanced	8.03%	7.77%	7.29%	7.02%	8.01%
CapStb	Cap Stable	6.43%	6.21%	5.54%	5.47%	6.34%
B	Bills	4.70%	4.70%	4.00%	3.96%	4.66%
D	Bonds	5.00%	5.00%	4.25%	4.23%	4.97%

18.2 For superannuation in the accumulation stage the "Balanced" portfolio compound average net investment return of 7.02% in the second last right-hand column of the above table, is indicative of the assumption that might be used in general-purpose deterministic calculations for superannuation products.

18.3 For superannuation in the pension stage the "Balanced" portfolio compound average net investment return of 8.01% in the right-hand column of the above table, is indicative of the assumption that might be used in general-purpose deterministic calculations for pension products.

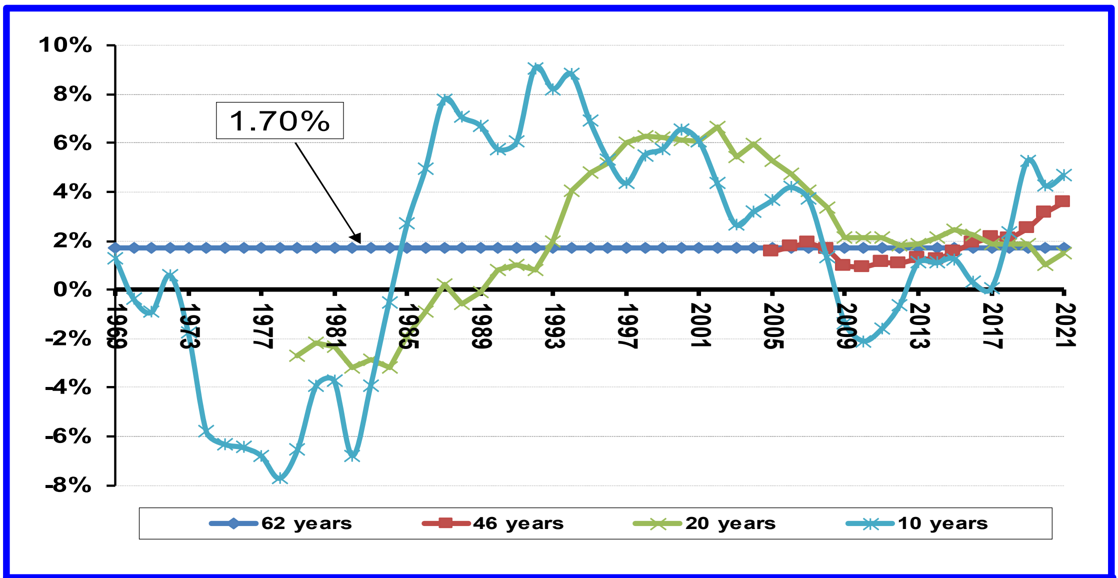
18.4 In Grenfell (2013) the Balanced" portfolio compound average rate in the second last right-hand column of the above table and the compound average rates for AWOTE and CPI in Table 16.1 were compared with the actuarial projections for long-term investment returns, wage growth and growth in the Consumer Price Index (CPI) reported by superannuation funds and published in APRA (2009). Unfortunately, APRA have not updated these actuarial projections so it is not possible to include a similar comparison in this paper.

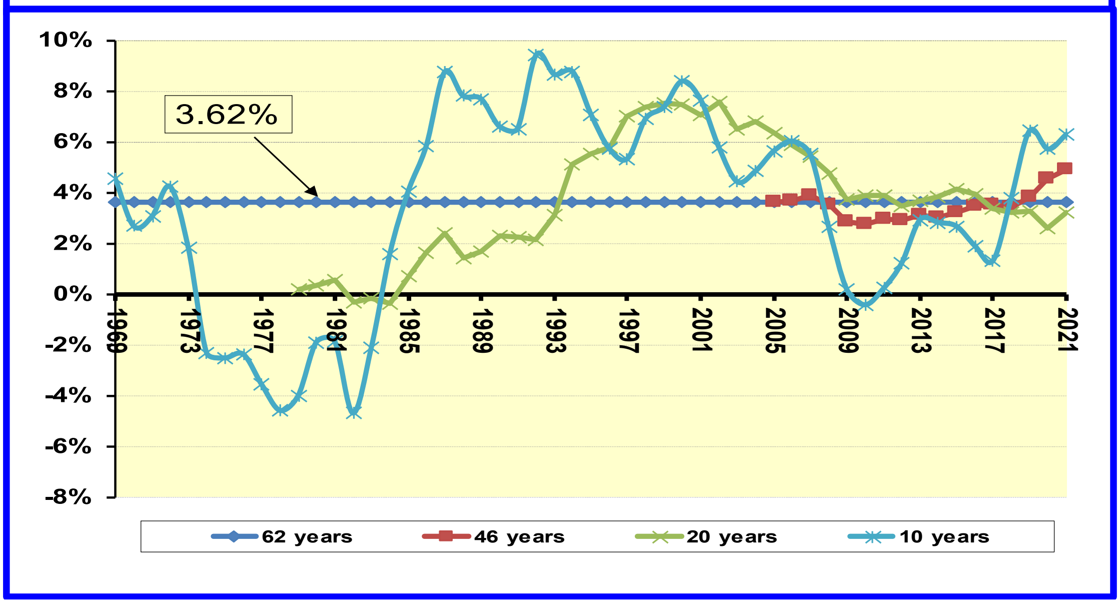
19 Further Observations

19.1 In this section all returns are net of tax and net of investment fees. The analyses allow for income tax at the current 15% superannuation accumulation stage rate and, on an approximate basis, for imputation credits and the lower rates of tax on realised capital gains after 12 months. The calculations assume that the current tax basis has applied at every year in the past, even though, in reality, tax on superannuation investment income did not commence until 1 July 1988.

19.2 Figure 19.1 shows compound average net annual real returns (over increases in AWOTE and, with yellow shading, over CPI) over 10, 20, 46 and 62 years ending 30 June 2021 for the Balanced portfolio described in section 8.3. The average AWOTE-based real annual return over 62 years was 1.70%. The average CPI-based real annual return over 62 years was 3.62%. The recent 20-year average real returns were slightly less than the 62-year average real returns because 20 years ago corresponds to near the Peak C referred to in section 9.6.

Figure 19.1 Balanced portfolio average net real returns





20 Concluding Remarks

20.1 As intended, this paper should help to bridge the gap between the demand from actuaries (and consumers) for robust assumptions in respect of future investment returns across a broad range of investment sectors, and the limited supply of data readily available for a range of investment sectors. The paper also demonstrates that the methodology appears to be robust with the additional data and remains suitable for the determination of justifiable and reasonable economic assumptions.

20.2 The advent of unit price data in Australia 56 years ago, through National Mutual's "EFG" system, supplemented with other published indices and rates, provided a substantial database of investment performance statistics across fifteen sectors, including eleven “asset” investment classes and four “liability” financial indicators.

20.3 The paper developed and summarized assumption sets for medium to long-term use in stochastic (and deterministic) investment models, across the range of investment sectors and economic indicators that were examined. In determining those assumptions, the paper has illustrated techniques for 'backdating' incomplete data series for use, particularly, with cross-correlation and auto-correlation analyses.

20.4 In addition, the paper adopted the technique used by Dwonczyk in fitting sine waves to CPI data, and applied an extended version of this technique to 10-year bonds and to CPI, AWOTE and Australian shares. This analysis appears to provide support for the existence of 'long-term' economic cycles extending over about 20 to 80 years (but much shorter and fluctuating for Australian shares) with the various estimates averaging about 46 years. A similar cycle length (45.0 years) was found in section 9.10 – this time using a different analysis of Balanced portfolio log “discounted” unit prices and measuring the periods between trend line minima. Economic cycles and auto-correlations appear to be areas that warrant further research.

20.5 The paper emphasises that wherever possible it is desirable for the setting of long-term assumptions to analyse results over at least one full economic cycle - and explains why 46-year periods were considered suitable for this purpose. Other observations include:

(a) care is needed when using running averages because their trends are impacted by both new data being introduced each year and by the old data dropping off (sections 2.7 and 2.9),

(b) the financial impact of the coronavirus global pandemic during 2020/2021 was far less severe than that of the 2007 global financial crisis (section 9.4).

(c) the impact of the global financial crisis was not an isolated event (section 9.5),

(d) the number of skewness and kurtosis assumptions that fall outside a minimal classification sounds a warning for those who use normal or lognormal models in relation to investment performance (section 12.4),

(e) although historically rates of AWOTE and CPI have had significantly different turning points from Bond rates, projections based on double sine curves show that turning points in future may coincide for all three rates with the next minima turning points being between December 2024 and March 2026 (sections 5.9, 5.13 and 14.6),

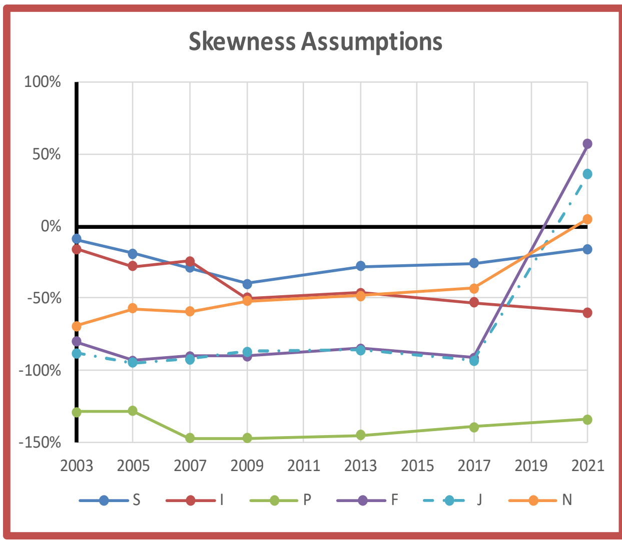
(e) there is a negative bias in Australian share auto-correlations up to a lag of 5.5 years, which illustrates the tendency for market prices to "bounce back" after falls and/or to "drop again" after price increases - somewhat similar, but different, to mean reversion (section 15.2),

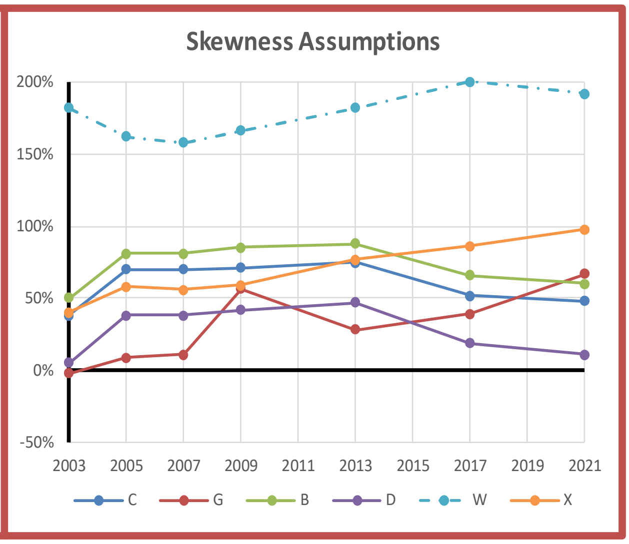
(f) for Australian shares, there is significant stability in S sector auto-correlations over the entire 62-year period examined (section 15.12),

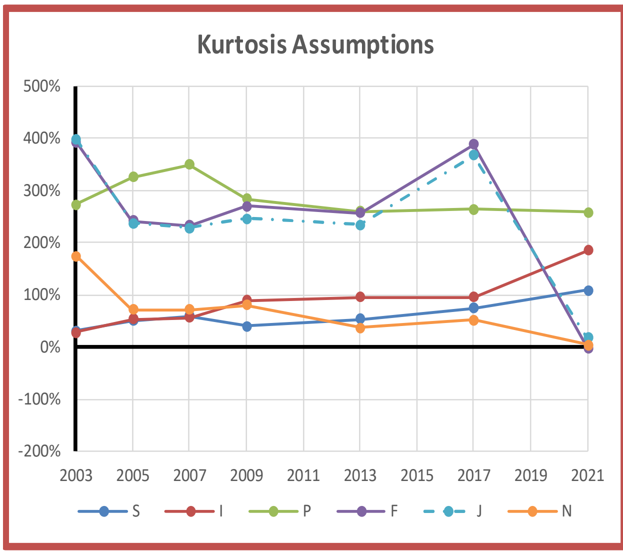
(g) for 10-year bond rates, 60-year auto-correlations have been reasonably stable over the 71-year period examined (section 15.12).

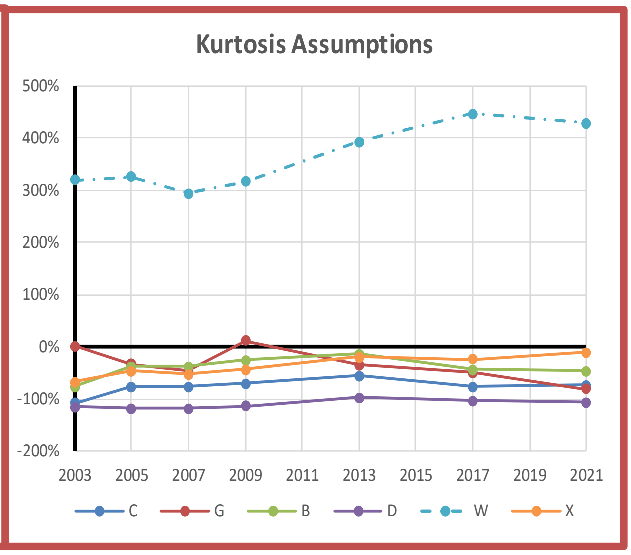
20.6 The methodology used to determine assumptions is explained in Section 2. With slight variations, such as for averaging periods and weights, that methodology was also used in 2003, 2005, 2007, 2009, 2013 and 2017. For 12 of the 15 sectors, Figure 20.1 compares the latest assumptions, for arithmetic means and standard deviations (before tax and fees) and for skewness and kurtosis, with the six previous assumptions.

Figure 20.1 Latest and previous assumptions









20.7 The reason for the significant changes in the F and J sector skewness and kurtosis assumptions between 2017 and 2021 is explained in section 11.7 (and also reflects the current shorter duration of the F sector mentioned in section 6.4).

20.8 Significant differences in the assumptions now recommended for correlations, as compared to those recommended four years ago, include:

		Now Recommended	**Previously Recommended**
Rank Cross-corr.	Sectors F and W	25%	17%
	F and X	18%	11%
	S and X	15%	8%
Auto-correlation	S @ lag 2 yrs	0%	-7%
	D @ lag 15 yrs	-12%	-34%

21 Acknowledgments

21.1 While I take full responsibility for the content of this paper, I would like to sincerely thank the following:

- Alan Brown who has graciously assisted for many years with guidance on statistics (including the bases underlying Appendix A), stochastic modelling (e.g. Appendices B and C), EXCEL programming and related topics.

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- Ord Minnett for their assistance with the provision of regular index values for six investment sectors (and my grandchildren and my wife for their assistance each quarter with checking the input of the latest data for the 15 sectors).

- The designers, in 1965, of the National Mutual EFG investment system.

- Ken Tan for his valuable assistance in chart preparation.

21.2 I am also indebted to Jeremy Waite for his article “ERM – Black Swans, Fat Tails and Spherical Cows” which was published in the April 2009 edition of *Actuary Australia*. Extracts from the article appear in various sections of this paper (in *italics*, within a border and under the heading “REMEMBER”).

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Appendix A: Modelling Skewness and Kurtosis

A1 This appendix describes a method for adding skewness (denoted g) and kurtosis (denoted k) to a long-term (say 40-year or 100-year) stochastic projection.

A2 Let x represent a uniform random variable (or “step”) with the values 0.5, 1.5, 2.5, 3.5 … 59.5. For the tails of distributions, steps of .0001, .001, .01 and .1 and 59.9, 59.99, 59.999 and 59.9999 were also considered.

A3 Let z represent a standardised normal random variable. For example, using an EXCEL function, z = NORMSINV(x/60).

A4 Applying the Normal Power Approximation the variable y given by:

y = z + g*(z^2-1)/6 + k*(z^3-3*z)/24 – (g^2)*(2*z^3-5*z)/36 [1]

will be found over the 60 steps from 0.5 to 59.5 inclusive to have a mean of approximately 0, standard deviation of approximately 1, skewness of approximately g and kurtosis of approximately k. By trial-and-error (or by using EXCEL Solver) it is possible to change the value of g to _g_ and the value of k to _k_ so that the variable _y_ (note _italics_) given by:

_y_ = z + _g_*(z^2-1)/6 + _k_*(z^3-3*z)/24 – (_g_^2)*(2*z^3-5*z)/36 [2]

has, over the 60 steps from 0.5 to 59.5, skewness of exactly g and kurtosis of exactly k. When the variable _y_ is standardised it will still have skewness of g and kurtosis of k.

A5 The Normal Power Approximation, applied as described above works well for those sectors with negative skewness and positive kurtosis and for those with low skewness and kurtosis. This group includes all sectors **where modest to significant negative returns are observed.** The forces for this group have positive kurtosis and include:

S, I, Q, P and H [section 3.1 explains the sector codes]

A6 However the above method is unsatisfactory (because the resultant variable does not increase for all steps from 0 to 60) for those sectors **where negative returns are not observed or where only occasional small negative returns are observed.** The forces for this group have negative kurtosis and include:

L, F, C, J, N, B, D and X [section 3.1 explains the sector codes]

A7 For this latter group it was found that instead of [1] above the following gamma exponential variable, y, expressed in EXCEL functions, could be used:

y = EXP(1-GAMMAINV(1-x/60,alpha,beta)) [3]

A8 Again, by trial-and-error (or by using EXCEL Solver) it is possible to change the value of alpha to _alpha_ and the value of beta to _beta_ so that the variable _y_ given by:

_y_ = EXP(1-GAMMAINV(1-x/60,_alpha_,_beta_)) [4] has, over the 60 steps from 0.5 to 59.5, skewness of exactly g and kurtosis of exactly k. When the variable y is standardised it will still have skewness of g and kurtosis of k. The resultant variable also increases for all steps from 0 to 60.

A9 Sector W (AWOTE) does not fit naturally into either of the two groups described on the previous page. It has high positive skewness and high positive kurtosis and (depending on the coefficient of variation assumed) can be expected to have occasional modest negative values. Interestingly, it was found in Grenfell (2013) that relationships [2] and [4] both produced satisfactory and almost identical values of y for this sector (except at the extreme tails). Relationship [4] was used for this update because the relationship [2] left hand tail variable did not increase as the step increased.

A10 Sector G (Government semis 0-3 years) sometimes does not fit naturally into either of the two groups described on the previous page. For this sector it was found that relationship [2] produced satisfactory values of y for the 2005 calculations but not for the 2007 and later calculations. For the latter four calculations, and for the current calculations, relationship [4] produced satisfactory values of y.

A11 The process described above produces standardised variables for all the 15 sectors with the desired skewness and kurtosis. However, this alone does not mean that the entire **shape of the distribution** of returns, and in particular the tail values, is realistic. To test this, the following analysis was performed:

(a) Sixty years of forces for each sector ending 30/9/19, 31/12/19, 31/3/20 and 30/6/20 were each **sorted and standardised** – giving four datasets.

(b) The skewness and kurtosis of each sector and dataset was calculated.

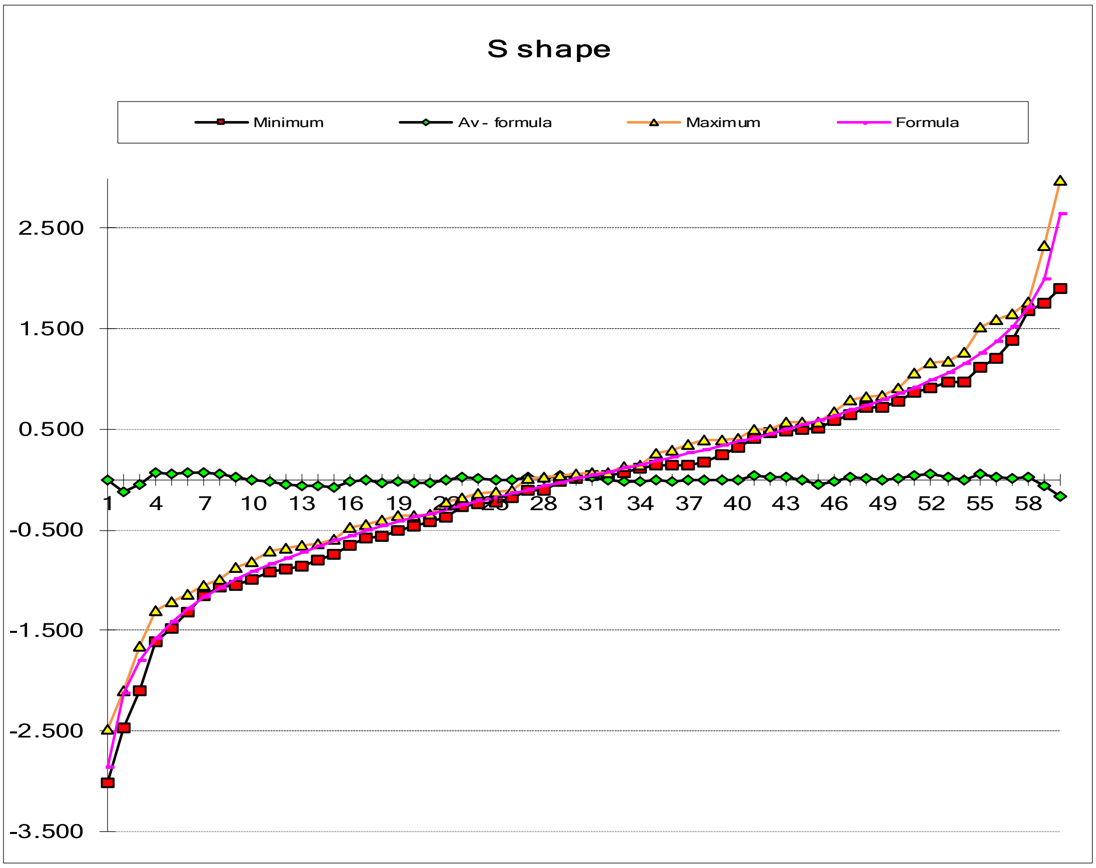
(c) The average skewness and kurtosis of each sector was calculated.

(d) Relationships [2] and [4] were applied based on the average skewness and kurtosis of each sector from (c) above, **not** based on long-term estimates of g and k.

(e) The resultant variables from (d) above (after standardisation) were compared with the four datasets from (a) above. The comparison focused, in particular, on whether the tail values for the resultant variables were:

- greater than the minimum of the tail values from the four datasets in (a) above, and
- less than the maximum of the tail values from the four datasets in (a) above.

(f) Two examples of where (e) was satisfied are shown in the following charts, one for the S (Australian shares) sector based on relationship [2], and one for the D (10-year bonds) sector based on relationship [4]. Each chart contains the maximum and minimum sorted standardised forces from the four datasets and also includes a graph (denoted "Av – formula") of the difference between the average of the four datasets and the formula result. A close study of the chart for the D sector will show that the relationship [4] formula values marginally exceeded the maximum extreme bottom left hand tail value, but they were judged as satisfactorily capturing the bottom left hand tail values when steps less than 23 were considered as a group.


S shape
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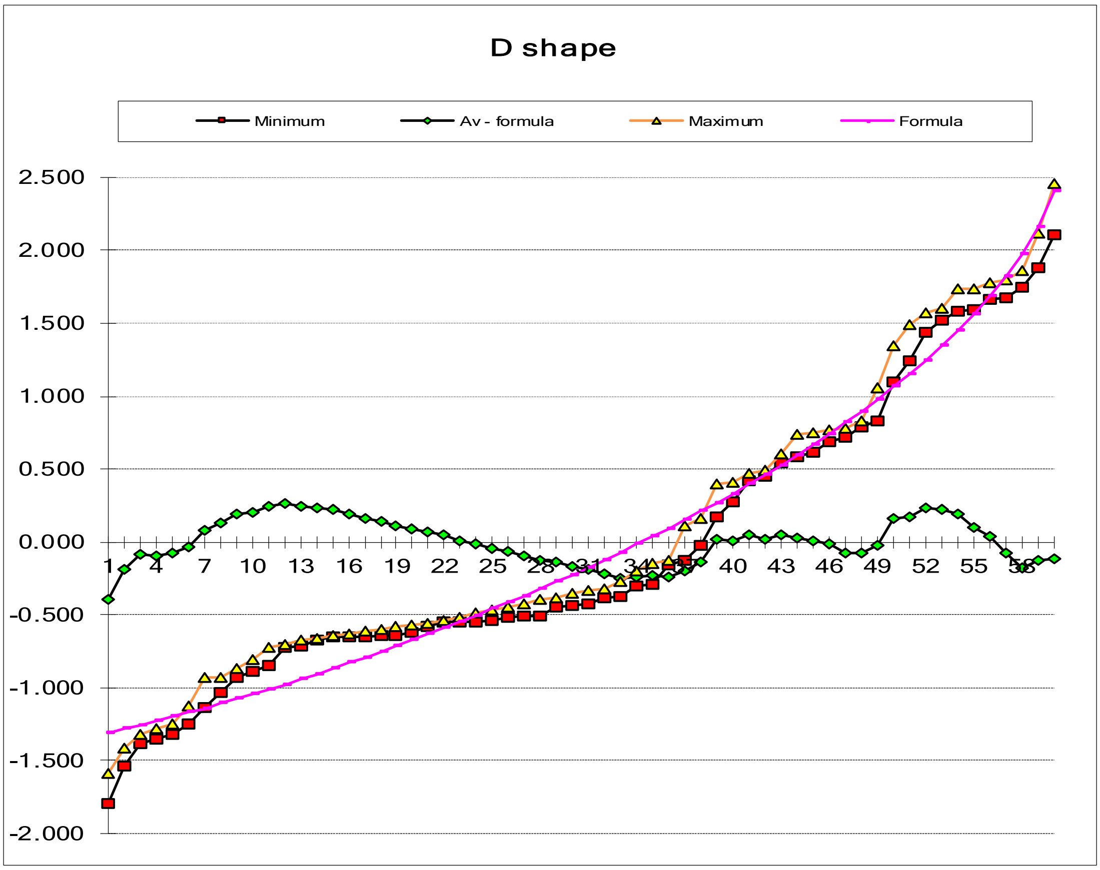
D shape
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-1.000
-1.500
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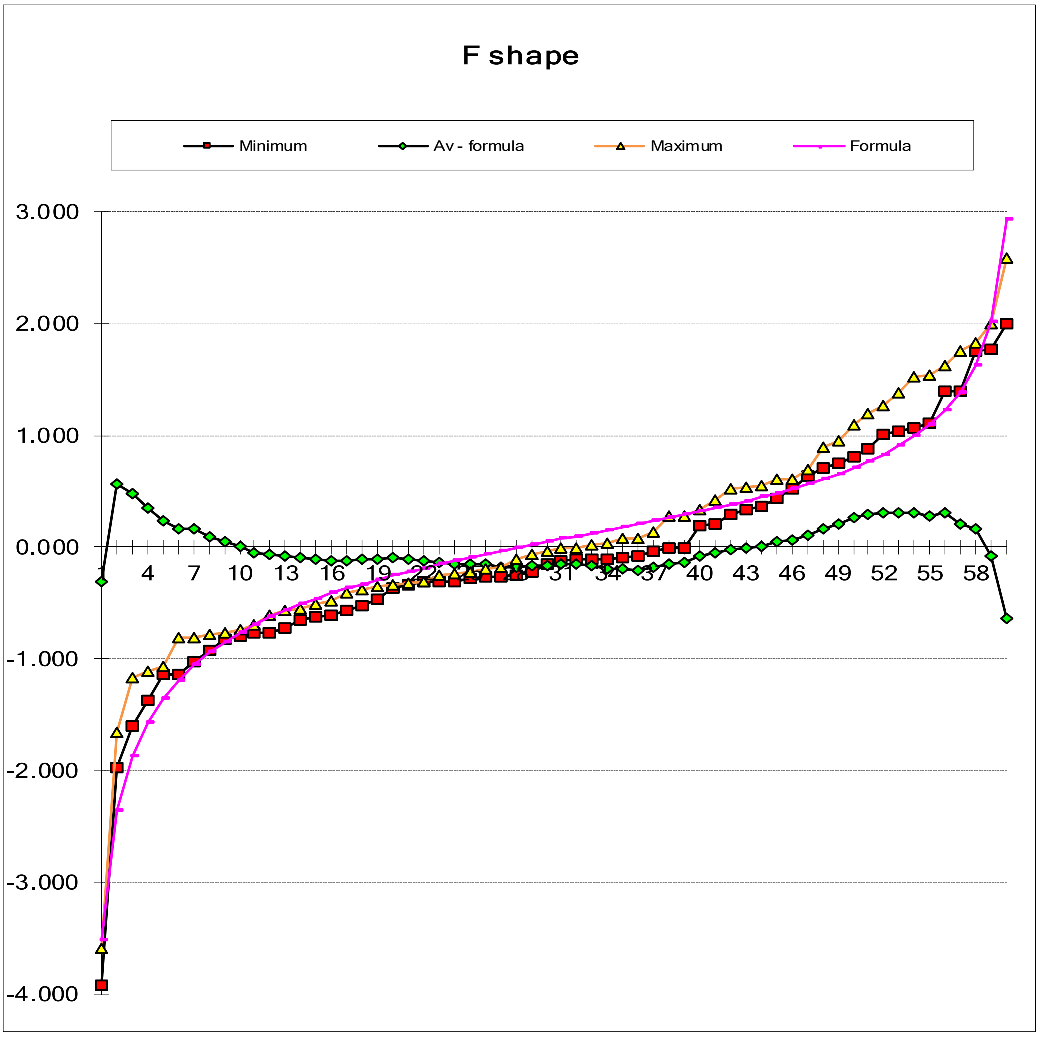
1 4 7 10 13 16 19 22 25 28 31 34 37 40 43 46 49 52 55 58

(g) An example of where (e) was not satisfied is shown in the following F (Australian fixed interest) sector chart.



A12 The chart in (g) above shows that relationship [2] has satisfactorily captured the bottom left hand tail value for the F sector. This same outlier is referred to in section 10.2. It relates to the year ending 30 September 1973. However, relationship [2] has not satisfactorily captured the top right-hand tail value for this sector since the formula result at point 60 exceeds the maximum. Because of the "backdating" formulae in Section 4, a similar problem occurred for the L, G, J and N sectors.

A13 To rectify the problem identified in the previous paragraph, it was found that an effective pragmatic solution was to reduce the skewness for the Q, L, F, G, J, C, N, B and X sectors by 0.14, 0.17, 0.20, 0.13, 0.20, 0.12, 0.14, 0.12 and 0.05 respectively **and** to reduce the kurtosis for the same sectors by three times the skewness reduction. The multiple of three recognises that an exponential distribution has a skewness of exactly 2 and a kurtosis of exactly 6 (being three times 2) and that the normal distribution fails to give correct probabilities for the tail of this distribution. With these reductions, relationships [2] and [4] satisfactorily captured both the bottom left hand and top right-hand tail values for the nine sectors and the shape for the non-tail values was insignificantly changed.

A14 The period chosen in A11(a) above was a suitable one because, except for the latest year Cash and Bill sector rates, it included, for all sectors and for the Balanced and Capital Stable portfolios, **245-year** (i.e. 1+ 61*4) extreme minimum and maximum values. This should therefore improve the accuracy of tail distributions. The dates at which some of the extreme returns occurred are listed in section 2.9. These 245-year extreme values are critical for the analysis in A11(b) to A13.

A15 The skewness and kurtosis assumptions tabulated in Table 16.1 are **gross** of (i.e. before) the reductions in A13.

A16 Relationship [4] has been shown to provide a good fit to the observed data for some sectors. However, care needs to be taken when applying these observations to asset risk management when the focus is on left hand tail distributions. In the last 62 years there has not been in Australia an occasion where a significant level of default on bonds or loans has occurred. It may be fallacious to assume that such an event will not occur in the future. Yet this is a (well-hidden) assumption within relationship [4].

REMEMBER

*We live in a non-stationary world, so using data to construct probability distributions can be dangerous… "**The idea that, for a tail distribution, we know enough about what's going-on to give a precise number in the tail is weird, frankly. …***

If you're going to insist on calculating a lot of numbers from the past, you need to make sure you don't think there is a risk of something happening in the future that hasn't happened in the period from which you are using data. …

"What are these numbers about? What are they for? What decision depends on them? Thinking those things through more deeply is worth a thousand numbers coming out of a black box model."

Lord Mervyn King (2020)

Appendix B: Modelling Auto-correlations

B1 An examination of Figure 15.1 shows that auto-correlations are of three broad types:

(a) Share sectors, Property Trusts and the "Balanced" portfolio

(b) Fixed interest sectors, Direct Property, CPI and AWOTE

(c) Bills and Bonds.

B2 Also of interest is the similarity between the auto-correlations of the S (Australian shares) sector and that of the "Balanced" portfolio. This is primarily due to the greater volatility of the S sector, which dominates the shape of the "Balanced" portfolio auto-correlations. Though not included in Figure 15.1, the auto-correlations of a "Capital Stable" portfolio are quite different from those for the "Balanced" portfolio. The "Capital Stable" portfolio auto-correlations are similar to those for the F sector.

B3 A closer examination of Figure 15.1 reveals that there is a gradual progression from the **S sector**, to Property Trusts, to the "Balanced" portfolio, to the F sector, to Direct Property, to CPI, to Bills and then to **Bonds**.

B4 **The auto-correlations of the S sector and Bonds form two extremes**. This hypothesis is supported by the following analysis of the auto-correlations underlying Figure 15.1:

i) for lags of 1 to 1.5 years, the S sector has the **lowest** auto-correlations of all the 15 sectors. In stochastic modelling the shorter-term lags are most important because there are far more observations at the short end than at the long end. Further, the longer results are to some extent repeats of the shorter results.

ii) for lags of 1 to 28 years, the D sector (i.e. Bonds) has the **highest** auto-correlations of all the 15 sectors.

iii) for lags of 14.75 and 15 years, 16.75 and 17 years and 28 to 28,75 years the S sector again has the **lowest** auto-correlations of all the 15 sectors.

iv) for lags of 31 to 32.25 years, 32.75 to 34 years and 32.25 to 35.5 years the D sector again has the **highest** auto-correlations of all the 15 sectors.

B5 Intuitively, the S sector is significant because of its higher volatility and expected investment returns. In addition, the D sector is significant because of its influence on current market investment yields.

B6 The above analysis forms the background as to why the *Austmod* stochastic investment simulation model focuses on the S and D sector autocorrelations.

B7 Briefly, the *Austmod* model sets D sector auto-correlations by sorting results into 48-year "bumpy" cycles as follows:

- subdividing 126 (i.e. 2*3*3*7) times, D sector zero auto-correlated stochastic results for 24 years out of 144 into six groups of 24 years,
- sorting each group of 24-year D sector results,
- 63 times, positioning each D sector result, first of first 24 to <u>first</u> of 48, second of second 24 to <u>second</u> of 48, third of second 24 to <u>third</u> of 48, fourth of first 24 to <u>fourth</u> of 48, etc., and

- retaining each result for the 14 other sectors with their original D sector value (thus leaving cross-correlations undisturbed).

B8 Briefly, the *Austmod* model then sets S sector autocorrelations by discarding all but one of the above D sector combinations, and retaining the D sector combination which is associated with the S sector auto-correlation closest to that desired. The method of least squares is used to determine the closest S sector auto-correlation, with the auto-correlations weighted towards the shorter lags (which have the greatest number of observations). The weight used is the square of (39 less the lag in years).

B9 Having obtained the S and D sector auto-correlations as above the (retained) cross-correlations carry through these auto-correlation distributions to the other sectors. The Cholesky decomposition formula is used to create the cross-correlation distributions.

B10 The process described in the previous paragraph produces auto-correlations for the remaining 13 sectors that lie between the S and D sector extremes and are primarily a function of their Table 16.3 rank cross-correlations with those for the S and D sectors. However, the results for X and W, where only about 55% of long-term historical auto-correlation distributions. In view of the D and X (0.69) and D and W (0.70) rank cross-correlations in Table 16.3, this result is not too surprising.

B11 By increasing the D and W and D and X cross-correlations by a further 0.12 (beyond which the Cholesky decomposition fails) it was found that about 80% of the desired auto-correlation distributions were obtained for both X and W.

B12 The results in Table 16.2 indicate that the D sector auto-correlation target is zero at a lag of about 13.4 years. When the *AustmodX* model is run on a "long-term" basis (with auto-correlation D sector cycles commencing at random positions) the results using 1000 simulations indicate a D sector auto-correlation of zero at a lag of about 12.0 years.

B13 When the *AustmodX* model is run with auto-correlation D sector cycles commencing at a fixed position, e.g. a position chosen to reflect current (low) 10-year bond rates, the results using 1000 simulations indicate a D sector auto-correlation of zero at a lag of about 11.6 years. Such simulations might be described as being based on "current financial conditions".

B14 The mid-point of the 12.0-year lag from B12 and the 11.6-year lag from B13 is 11.8 years. Recognising that a complete D sector auto-correlation cycle is about 4 times the lag where auto-correlations first reduce to zero, consideration of B13 and B14 above suggests that if the 48-year cycle in B7 above were increased by 4*(13.4 -11.8) = 6.4 or by say 6 years, it should produce satisfactory D sector auto-correlations.

B15 For further information about the appendices, the historical database or *Austmod*, the author can be contacted at colin.grenfell2@outlook.com.

Appendix C: The *Austmod* Investment Simulation Model

C1 *Austmod* is a stochastic and historical investment simulation model. The model is an EXCEL workbook that displays up to 62 years of historical and 40 or 100 years of simulated investment performance for the 15 "sectors" described in Section 3.

C2. This appendix describes the input, algorithms and output for the latest versions of the model (version X based on the "OLD" assumptions and version Y based on the "NEW" assumptions).

C3 The 34 inputs to the model are:

Which	What	Input
TAX	0 = Nil 1 = Ordinary 2 = Superannuation 3 = Exempt 4 = Others	[1]
	40 years? [2] Enter number of years for which the above tax basis applies	[3]
	(Y =yes, N =no) Enter new tax basis (0, 1, 2, 3 or 4) for the remainder of 100 years)	[4]
MODEL	Type : 0 = stochastic 1 = historical random start date	[5]
	Start seed : Enter a number from 1 to 32000 (or enter R for random)	[6]
PPNS	**Balanced >>** 32% Shares	
	22% Int' Shares (uh)	
	Usually **Composite** Press 7% Property Trust	
	enter 0% : **portfolio** F9 if 3% Property Direct	
	(enter 0, 1, 2 or 3) : 'manual' 0% Loans/ credit	
	Bill Rate [8] 7% Int'l Shares (h)	[9]
	Bond Rate [7] **0 = Input primes** 12% Fixed Interest	
	AWOTE **1 = Capital stable** 0% Semi Government	
	CPI **2 = Balanced** 7% International Bonds	
	3 = Growth 8% Cash	
	2% Inflation Linked	
PLAN	Assets at start $	[10]
	Liabilities (or accumulated retirement benefits) at start $	[11]
	Annual net Enter the net cash flow per annum at start (as a number)	[12]
	cash flow : ~ varies W = ~ AWOTE X = ~ CPI $ = dollar P = % assets	[13]
	Driver: A=3yr geo av B=bills C=cash D=bonds E=earnings W=AWOTE	[14]
	Crediting rate formula: 1 = standard 2 = driver 3 = other	[15]
INVEST	**Means: 1 = based on standard 2 = other**	[16]
	Additional return: (all sectors except W and X, enter additional rate before tax	[17]
	Net of fees: 0 = no 1 = standard 2 = other	[18]
	Standard deviations: 0 = nil 1 = standard 2 = other	[19]
	Skew & kurtosis: 0 = nil 1 = std (skew only) 2 = std (both)	[20]
	Cross correlations: 0 = nil 1 = standard 2 = std+WX 8 yr lags 3 = other	[21]
	Auto correlations: 0 = nil 1 = C cycles 2 = S+D trends	[22]
	Seasons start for D: Enter number of years since bottom (0=bot, 24=top, R=random)	[23]
	Historical results: Enter number of years (between 1 and 62)	[24]
	Historical results ending: Enter 31/ 3, 30/ 6, 30/ 9 or 31/ 12 dates only	[25]
INITIAL	Fund earning rate in the year prior [25] (= assumed start date)	[26]
	Fund earning rate in 2nd year prior [25] (needed for driver)	[27]
	Nominal bill rate at start (if Driver = B and end or start year based)	[28]
	Nominal bond rate at start (if Driver = D and end or start year based)	[29]
2ND	**Balanced >>** 36% Shares	
	19% Int' Shares (uh)	
	Usually **Composite** Press 7% Property Trust	
	enter 0% : portfolio F9 if 2% Property Direct	
	(enter 0, 1, 2 or 3) : "manual' 0% Loans/ credit	
	Bill Rate [31] 6% Int'l Shares (h)	[32]
	Bond Rate [30] **0 = Input primes** 16% Fixed Interest	
	AWOTE **1 = Capital stable** 0% Semi Government	
	CPI **2 = Balanced** 7% International Bonds	
	3 = Growth 5% Cash	
	2% Inflation Linked	
2ND	**Annual net** Enter the net cash flow per annum at 2nd Tax start (as a number)	[33]
	cash flow : ~ varies W = ~ AWOTE X = ~ CPI $ = dollar P = % assets	[34]

C4 Inputs [1] to [34] are briefly explained below.

TAX (Taxation)

[1] Enter 0 for no taxation and no imputation credits.
Enter 1 for life company "Ordinary" taxation.
Enter 2 for superannuation tax (in the accumulation stage).
Enter 3 for exempt from taxation but with imputation credits (for example, pensions and annuities).
Enter 4 for another taxation basis (the "watch" column, not illustrated above, will then indicate where the new taxation basis is entered).

[2] Enter "Y' if projections are for 40 years
Enter "N" if projections are for 100 years (i.e. not for 40 years)

[3] Enter the number of years for which the tax basis from [1] above applies

[4] Enter 0, 1, 2, 3 or 4 (see above) to indicate the tax basis for the remainder of 100 years.

MODEL (Modelling basis)

[5] Enter 0 for stochastic modelling.
Enter 1 for "historical random start" modelling. With this basis, historical data for the period specified in [24] and [25] of C3 on the previous page will be stacked end-on-end to form a recurrent sequence and then 1, 8, 40, 200 or 1000 40-year or 100-year scenarios can be reported from this sequence, each based on a random start date. This is a form of bootstrapping – the following extract from Sweeting (2007) explains:

"In stochastic modelling the first distinction is between bootstrapping and forward-looking approaches. For bootstrapping, all that is needed is a set of historical data for the asset classes being modelled. For example, monthly market returns for the last 20 years could be used. However rather than simply using this as a single 'run' of data, modelling is carried out by selecting a slice of data randomly, in this case the results from a particular month. This forms the first observation. This observation is then 'replaced' and another month is chosen randomly. This means that a relatively small data set can be used to generate a large number of random observations.

The main advantage of bootstrapping is that the underlying characteristics of the data and linkages between data series are captured without having to resort to parameterisation."

Unlike bootstrapping, "historical random start" (or "HRS") modelling captures most of the auto-correlation of the historical data as well as most of the cross-correlation. For the chosen period (specified in [24] and [25]) and for all sectors, all of the historical auto-correlation and cross-correlation is captured except for when one sequence stops and another starts. Further information and an example of HRS modelling is contained in Sneddon (2019).

[6] Enter a "seed" number between 1 and 32,000 or enter "R" for a random series.

PPNS (Proportions)

[7] For bills, bonds, AWOTE and CPI the default 0% proportion will normally apply since it is not possible to "invest" in these sectors. However, for model-testing it may be desired to enter 100% in one of these cells.

[8] Enter 0 if proportions for "prime" investment sectors are being entered in [9] below.
Enter 1 if the default "Capital Stable" portfolio proportions are to be used.
Enter 2 if the default "Balanced" proportions (illustrated in C3 above) are to be used.
Enter 3 if the default "Growth" portfolio proportions are to be used.

[9] If the default composite portfolio proportions are not being used, enter the desired proportions for each sector. These proportions must total 100%.

PLAN (Plan details]

[10] Enter the market value of assets at the projection start date (i.e. at the date [25]).
[11] Enter the liabilities or the accumulated retirement benefits at the projection start date.

[12] Enter the annual net cash flow at the projection start date.

[13] Enter "W" if the annual net cash flow varies proportional to AWOTE changes.
Enter "X" if the annual net cash flow varies proportional to CPI changes.
Enter "$" if the annual net cash flow is a constant dollar amount.
Enter "P" if the annual net cash flow is a proportion of plan assets (the "watch" column, not illustrated, will then indicate where the required proportion/s is/are entered).

[14] The "driver" is a component of the crediting rate basis or of the growth in liabilities.
Enter "A" if the driver is the geometric 3-year average of past plan investment returns.
Enter "B" if the driver is based on the Bill rate.
Enter "C" if the driver is based on the Cash investment return.
Enter "D" if the driver is based on the Bond rate.
Enter "E" if the driver is the plan investment return (earnings).
Enter "W" if the driver is based on changes in AWOTE.

[15] Enter 1 if the crediting rate formula is "standard" (this is unlikely to apply but a standard formula is built into the model which can be used, varied or ignored).
Enter 2 if the crediting rate equals the driver.
Enter 3 if neither 1 nor 2 applies (the "watch" column, not illustrated, will then indicate where the "other" basis is entered – similarly, when "other" is chosen for [16], [18], 19] or [21] below, the "watch" column will indicate where the basis is entered).

INVEST (Investment assumptions)

[16] Enter 1 if the mean is "standard" (see Table 16.1).
Enter 2 if the assumption for the mean is "other" (note the "watch" column).

[17] Enter any "additional return" (before tax) to be added to, if positive, or subtracted from, if negative, the mean from [16] above (for all investment sectors).

[18] Enter 0 if investment returns are to be before investment fees.
Enter 1 if investment returns are to be net of "standard" investment fees.
Enter 2 if investment returns are to be net of "other" investment fees.

[19] Enter 0 if the standard deviation of all returns is zero (in which event all projections will be "deterministic").
Enter 1 if the standard deviation is "standard" (see Table 16.1).
Enter 2 if the standard deviation is "other".

[20] Enter 0 if the skewness and kurtosis of all returns is zero (in which event all projections will be based on log-normal distributions).
Enter 1 if L, C, B, D, W and X sector returns (only) are to be skewed on a "standard" basis so as to avoid negative returns (or significantly negative returns for W and X sectors) and for the 8 other sectors log-normal distributions are required.
Enter 2 if the skewness and kurtosis of all returns is "standard" (see Table 16.1 and Appendix A).

[21] Enter 0 if the cross-correlations of returns are zero (i.e. independent).
Enter 1 if the cross-correlations of returns are "standard" (see Table 16.3 and section 13.4).
Enter 2 if the cross-correlations of returns are "standard" with an 8-year lag for W and X sectors (see Section 14.7). This option has been retained but is now not recommended.
Enter 3 if the cross-correlations of returns are "other".

[22] Enter 0 if the auto-correlations of returns are zero (i.e. serially independent).
Enter 1 if the auto-correlations of C sector returns are based on a fixed 4-year cycle.
Enter 2 if the auto-correlations of S and D sector returns are based on the method explained in Appendix B.

[23] When 1 or 2 is entered in [22] above, the returns for the C or D sector will be based on cycles (also referred to as "seasons"). The code entered in [23] then determines when these cycles start. If 1 is entered in [22] then:

Enter D if the cycle starts with a downward movement (half-way from top to bottom).
Enter B if the cycle starts at the bottom.
Enter U if the cycle starts with an upward movement (half-way from bottom to top).
Enter T if the cycle starts at the top.
Enter R if the cycle starts from a random position.

Alternatively, if 2 is entered in [22] then:
Enter 0 if the D sector sine curve cycle starts at the bottom, or otherwise
Enter the number of years from the bottom to the desired start, or
Enter R if the cycle starts from a random position.

[24] Enter the number of years (between 1 and 62) over which historical returns are required.

[25] Enter the end-date (31 March, 30 June, 30 September or 31 December dates only) for historical returns. This date is also the start date for projections. The date must be between 30/6/1960 and 30/6/2021 inclusive.

INITIAL (Initial information)

If further information is unnecessary, the description for [26] to [29] will be blank and any input will be irrelevant (as explained, but not illustrated, in the "watch" column). In most situations the input in [11] will **not** require further "INITIAL" information.

[26] Enter the fund earning rate in the year prior [25].
[27] Enter the fund earning rate in the year prior to the year prior [25].
[28] Enter the nominal Bill rate at the date in [25].
[29] Enter the nominal Bond rate at the date in [25].

SECOND (2nd Investment Proportions and Cash Flow basis)

If [2] above specifies that 40-year projections are required, then the description for [30] to [34] will be blank and any input will be irrelevant (as explained, but not illustrated, in the "watch" column).

If [2] above specifies that 100-year projections are required, then enter the "second" investment proportions in [30] to [32] (as explained for [7] to [9] above) and the "second" cash flow basis in [33] and [34] (as explained for [12] and [13] above).

C5 The model has an annual time scale. The mathematical structure of the underlying *AustmodX* and *AustmodY* algorithms is summarized below.

Random numbers. If seeds are used, random numbers are "controlled" by selecting 2,000 random numbers (8 times), then calculating a second set of numbers equal to one minus these and mixing the 16 sets together so that all 32,000 have near zero auto-correlation.

Normal. First, the model generates independent Normal random variables for each sector.

Cross-correlation. These random variables are then converted to dependent Normal random variables using the *Cholesky* decomposition formula (refer Wilkie A D, 1988, JIA 115. Part 1, page 51).

Skewness and kurtosis. These random variables may then be converted to dependent non-Normal random variables using formula [2] or [4] as described in Appendix A.

Shape. For sectors Q, L, F, G, J, C, N, B and X, the shape of the distribution may then be improved by slightly reducing both the skewness and kurtosis - refer Appendix A, paragraph A13.

Taxation. The input for each sector includes two tax rates and information for imputation credits. The *income tax rate* is applied to the long term expected income yield and imputation credits. The *deferred tax rate* is applied to the total before tax yearly return less the long term expected income yield. This is equivalent to assuming, for tax purposes, that all fluctuations in investment returns are due to fluctuations in the capital appreciation component. The *after-tax standard deviation* for each sector equals the *before-tax standard deviation * (1 - deferred tax rate)*.

Additional return. The before tax standard investment return for each investment sector may be increased (or decreased if negative) by adding an *additional return*. For tax purposes the *additional return* is treated as a capital appreciation component and taxed at the *deferred tax rate*.

Investment fees. The *investment fee* for each sector is deducted from the before tax long term expected income yield; the result is then taxed at the *income tax rate*.

Forces. After allowing for taxation and any additional return and investment fees, the standardized random variables (denoted *srv*), are then converted to annual forces using the formula *force = mu + sigma*srv*.

Mixture. The annual force for the mixture or portfolio is then determined by weighting the sector forces by the proportions for each sector. The proportions may be specified individually, or default proportions for "Growth", "Balanced" or "Capital Stable" portfolios may be used.

Rates. The annual forces are then converted to rates using the formula
rate = EXP(force) - 1.

Repeats. The above 10 steps are repeated 164 times to give a 164 single-year scenarios with no auto-correlation.

Auto-correlation. 48-year scenarios with auto-correlation may then be generated using the methodology described in Appendix B, paragraphs B7 to B10.

Lags. CPI and AWOTE may then be lagged (but refer Section 14.7).

Refinement. The CPI and AWOTE auto-correlations may be further improved by increasing their cross-correlation with the D sector by 0.12 (refer paragraph B11).

C6 The output for *Austmod* includes the following:

Investment Output	Historical and simulated annual returns (and their ranks) for the 15 sectors and for the mixture or portfolio
Investment Return Charts	Historical and simulated annual returns and auto-correlation charts
Plan Output	Output for a superannuation or other plan, product or member showing: (1) accumulated retirement benefit at end of each year, (2) price-deflated accumulated retirement benefits, (3) wage-deflated accumulated retirement benefits, (4) accumulated retirement benefits as a multiple of salary, (5) equivalent retirement benefit scale, (6) decumulation account balance at end of each year, (7) wage-deflated decumulation account balances, or (8) reserve ratios (F-A)/A at end of each year
Plan Charts	Various charts of the plan output and distributions
Crediting Rate Charts	Charts of simulated crediting rates and investment returns
Research Output for 40-year projections	Output for a superannuation or other plan, product or member showing: (1) mixture annual returns each year, (2) L, C, B, D, W, and X minima and maxima, (3) S sector force auto-correlations for lags 1 to 38 years, (4) D sector force auto-correlations for lags 1 to 38 years, (5) force cross-correlations (samples – assumptions),

	(6) decumulation annual drawdowns, (7) wage-deflated decumulation annual drawdowns, or (8) LN fund ratios LN (F/A) at end of each year
Research Output for 100-year projections	Output for a superannuation or other plan, product or member showing: (1) mixture annual returns each year - first tax and mix, (2) mixture annual returns each year - second tax and mix, (3) mixture annual returns each year - both taxes and mixes, (4) L, C, B, D, W, and X minima and maxima, (5) force cross-correlations (samples – assumptions), (6) decumulation annual drawdowns, (7) wage-deflated decumulation annual drawdowns, or (8) LN fund ratios LN (F/A) at end of each year
Research Chart	Various charts of research output
Skewness and Kurtosis	Worksheet of Appendix A, paragraph A1 to A16 calculations
Assumptions and Analysis	Comparisons of mean, standard deviation, skewness, kurtosis, rank cross-correlation and auto-correlation assumptions with the average simulated results for each sector and mixture across 8, 40, 200 or 1000 simulations, and taxation calculations.
Sector Long Term Estimates	Long term estimates and taxation calculations for "Ordinary" (Life Insurance), "Superannuation" (in the accumulation stage), "Exempt" (pensions or annuities) and "Nil tax".
Actual Forces (Gross of Tax)	Historical annual forces at quarterly intervals from 30/6/1960 to 30/6/2021 and auto-correlations
Macros	Eight macros for simulations, correlations, "season" calculations, menu, plan and research output options, random number and random seed calculations (seeded minimum/maximum event probabilities are one in 50,000)

REMEMBER

> *"**There is a great deal of belief in mathematical models – though some models are useful, all models are wrong.***
>
> *This is not to say models aren't helpful, but that they need to be combined with experience, business acumen and judgement, and ... used properly. It is important to know what is not included. They also have a strong part to play in 'what-if' analysis. ...*
>
> *Models have a useful part to play and clear communication of the parts of the risk that are not modelled is crucial information in order to be able to use the models as a useful input to making strategic decisions. ...*
>
> *The communication skills of the actuarial profession have never been more needed."*
>
> *Waite (2009)*